TOPIC 1.2: SLOPES OF LINES

PERFORMANCE OBJECTIVES

Students will be able to:

- identify the slope of a line from a graph or an equation
- recognize parallel and perpendicular lines graphically and algebraically
- prove that two lines are parallel iff (if and only if) their slopes are equal and that two lines are perpendicular iff their slopes are negative reciprocals

MATERIALS

Overhead projector, prepared transparencies

STRATEGIES

- As a Do Now for this lesson, ask students to graph the following two pairs of equations on coordinate axes. Ask them how are the lines geometrically related

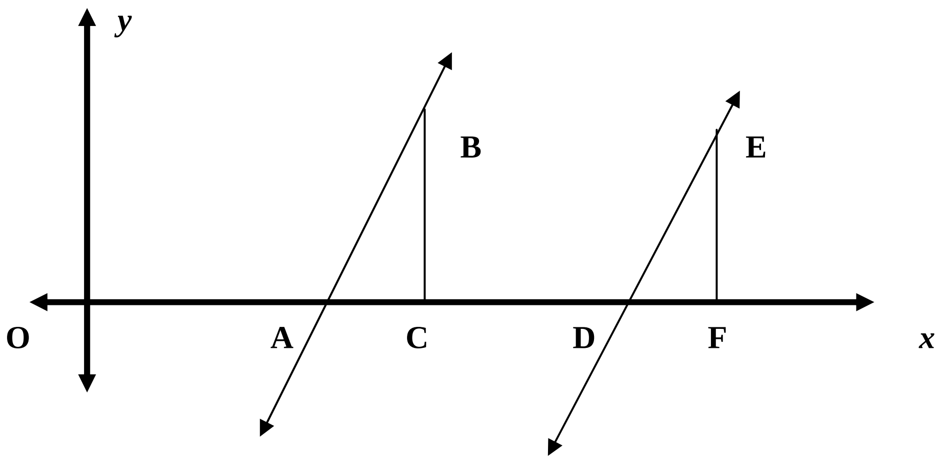
  (a) $3x + 2y = 6$ and $y = -\frac{3}{2}x - 3$

  (b) $x - y = 4$ and $x + y = 4$

- Elicit from the class that in (a) of the do-now, the lines are parallel since their slopes are equal. Further elicit from the class that in (b) of the do now, the two lines are perpendicular since their slopes are negative reciprocals. Discuss the importance of parallel and perpendicular lines in real life applications.

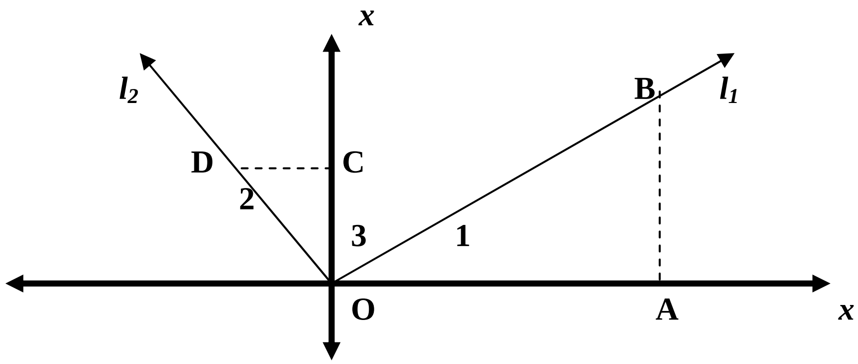
- Summarize the previous discussion on the board, including the definition of slope, the slope intercept form $y = mx + b$, that two non-vertical lines are parallel if and only if they have the same slope, and that two lines are perpendicular if and only if their slopes are *negative reciprocals* of each other.
- Suggest to the students that the proof of these two "iff" theorems were stated but usually not proven in earlier courses. It is appropriate to include the proof(s) of these theorems in a statement-reason format. Have one or both (if time permits) of the following proof(s) written on an overhead transparency and discuss it with the class. The proof of "two lines are perpendicular iff their slopes are negative reciprocals of each other" may also be prepared on a transparency and presented to the class.

- Practice the application of these two theorems by having the class write the equation of a line perpendicular to $3x - 2y = 12$ and passing through (-1, 6). Another example that may be posed is: Find two points on the line $x - 3y = 6$ and show that the slope of the line passing through these two points is the same as the slope m when $x - 3y = 6$ is rewritten in the standard form $y = mx + b$.



STATEMENTS	**REASONS**
(1) AB // ED	(1) Given
(2) ∠ BAC ≅ ∠ EDF	(2) If two parallel lines are cut by a transversal, the pair of corresponding angles formed are congruent.
(3) Drop ⊥ from B to AC Drop ⊥ from E to DF	(3) One and only one perpendicular may be drawn from a given point to a given line.
(4) ∠ BCA and ∠ EFD are right angles	(4) Perpendicular lines form right angles.
(5) ∠ BCA ≅ ∠ EFD	(5) All right angles are congruent.
(6) Δ BAC ~ Δ EFD	(6) AA theorem
(7) $\frac{BC}{AC} = \frac{EF}{DF}$	(7) Definition of Similar Triangles
(8) $m_{AB} = m_{DE}$	(8) Definition of slope

The converse of this may be proven by reversing the steps and using the SAS similarity theorem.



STATEMENTS | **REASONS**

STATEMENTS	REASONS
(1) Drop ⊥ from D to line OC and from to line OA	(1) One and only one line may be drawn perpendicular from a given point to a given line.
(2) Slope l_1 • Slope l_2 = -1	(2) Given
(3) Slope $l_1 = \frac{AB}{OA}$ and Slope $l_2 = -\frac{OC}{DC}$	(3) Definition of Slope
(4) $\frac{AB}{OA} = \frac{DC}{OC}$	(4) Substitution Postulate (step 2,3)
(5) ∠ DCO and ∠ BAO are right angles	(5) Perpendicular lines form right angles.
(6) ∠ DCO ≅ ∠ BAO	(6) All right angles are congruent.
(7) Δ DCO ≅ Δ BAO	(7) SAS Similarity
(8) ∠ 1 ≅ ∠ 2	(8) Definition of Similar Triangles
(9) ∠ 1 + ∠ 3 form a right angle and = 90°	(9) The coordinate axes are perpendicular and perpendicular lines form right angles.
(10) ∠ 2 + ∠ 3 form a right angle and = 90°	(10) Substitution Postulate
(11) $l_1 \perp l_2$	(11) Definition of Perpendicular

This proof is optional depending on time.

Lesson plan by Craig D. Smith