TOPIC 1.3: FINDING EQUATIONS OF LINES

PERFORMANCE OBJECTIVES:

Students will be able to:

- write the equation of a line given various geometric properties, such as the slope, the intercepts, and/or the solutions of the linear function
- write the equation of a line given other lines that are either parallel or perpendicular to that line
- write the equation of a line in several different forms: the intercept form $\frac{x}{a} + \frac{y}{b} = 1$, the slope-intercept form $y = mx + b$, and the point-slope form $y - y_1 = m(x - x_1)$
- use the equations of a line to prove or show certain features of various geometric figures

STRATEGIES (This lesson may require two days. One day may be used for the development and one day for applications.)

- Use the following Do Now to get this lesson started:

  (a) On graph paper, graph $\frac{x}{8} + \frac{y}{4} = 1$.

  (b) Find the x- and y-intercepts of this graph.

  (c) How are the intercepts related to the graph's equation?

- From the "Do Now" introduce the intercept form of a line $\frac{x}{a} + \frac{y}{b} = 1$. Students will be able to see the intercepts from the graph and will make a connection between this form of the equation and the graph.

- Use the graph and the intercept form to elicit the slope-intercept form $y = mx + b$. They already have the y-intercept and can generate the slope from the graph. This is the form with which the students will be most familiar.

- Have the students write an equation of a line passing through (2, 3) and having a slope of $-\frac{2}{3}$.

  From this discussion, elicit the point-slope form of the line $y - y_1 = m(x - x_1)$. In order to help elicit the set up of this equation, have students discuss how they obtained the slope and how it is possible to get the slope given solutions of the graph.

- Summarize the discussion by listing the three standard forms of a straight line on the board.

  Slope-intercept form $y = mx + b$ [line has slope "m" and y-intercept "b"]

  Point-slope form $y - y_1 = m(x - x_1)$ [line has slope "m" and contains (x_1, y_1)]

  Intercept form $\frac{x}{a} + \frac{y}{b} = 1$ [line has x-intercept "a" and y-intercept "b"]

- Pose the following five problems to the class and ask them to identify the most appropriate standard form of a linear function to use for each:
  Write the equation of the linear model described below:

  (a) The line with a slope of -2 and passing through the point (3, 4).

  Answer: $(y - 4) = -2(x - 3)$

  (b) The line passing through the points (-1, 4) and (5, 8).

  Answer: $(y - 4) = \frac{2}{3}(x + 1)$ or $(y - 8) = \frac{2}{3}(x - 5)$. Verify that these two lines are the same line.

  (c) The line with an x-intercept of -1 and passing through the point (0, 6).

  Answer: $\frac{x}{-1} + \frac{y}{6} = 1$

  (d) The line that goes through the point (-2, 4) and that is parallel to the line that passes through (1, 1) and (5, 7).

  Answer: $y - 4 = \frac{3}{2}(x + 2)$

  (e) The line that is the perpendicular bisector of the segment joining A (-2, 3) and B(4, -5).

  Answer: $y + 1 = -\frac{4}{3}(x - 1)$

- Summarize the basic concepts of the lesson by posing the following:
  Write, in three different forms, the equation of a line with a slope of $-\frac{1}{2}$ and an x-intercept of -6.

- If time permits, pose and discuss the following problem. In the discussion, discuss the fact that the angle bisector is the median of an isosceles triangle.
  Triangle DEF has vertices D(4, -1), E(2, -1), and F(3, 6).
  (a) Verify that triangle DEF is isosceles.
  (b) Write an equation of the bisector of angle F.
  In this example, the angle bisector is a vertical line x = 3. Review the equation of vertical lines and the fact that vertical lines have no slope.

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