TOPIC 1.4: LINEAR FUNCTIONS AND MODELS

PERFORMANCE OBJECTIVES

Students will be able to:

- model real life situations using linear functions
- write a linear function that describes a rule from a given table of data
- write a linear function from a verbal example

MATERIALS

Graphing calculator, graph paper, overhead projector

STRATEGIES (This lesson may require two days.)

- Pose the following problem to the students to get the lesson started:
  Two college students decided to earn money by starting a catering business. They spent $50,000 to start the business. Now they earn $5,000 for each event they cater.

  (a) Complete the following table:

Number of events (n)	0	1	5	10	15	30
Net income (I)	-$50,000	-$45,000				

  (b) From the table, graph the data on a coordinate axes. Be sure to label the scale of the graph and the x and y intercepts.
  (c) From the graph determine the equation of this line.

- From the first problem, students should be able to fill in the blanks of the table, plot these points on a set of coordinate axes, and determine the equation produced by this data (using slope and y-intercept). Discuss with the class the importance of labeling axes. The x-axis for this problem could be one box equals 5 events. The y-axis could be one box equals $5000. If students struggle with finding the slope on the graph, they could use two data points from the table to determine the slope algebraically and then the equation. From the labeling of the axes, some students may think that the slope is either 2.5, because rise is 2.5 and run is 1. Discuss the importance of using the definition of slope. The y-intercept is –50,000.

- Have the class express the equation of the line from the first problem in words and then using symbols. "Net income equals their fee per event times the number of events minus their initial investment" and then "I = 5000n - 50,000" From the graph determine the domain and range of this function. Explain what real values are left out of the domain and range and why. Summarize that the domain is n > 0, for *n* an integer, and that the range is I > -50,000. Further summarize that negative numbers are left out of the domain because it is not possible to cater a negative number of events. The range cannot go below -50,000 because that is as far in debt as they can be.

- Using the catering problem review the terms *dependent* and *independent* variables. The net income is dependent on the number of events catered. Therefore, net income *I*, is the dependent variable and number of events *n*, is the independent variable. Elicit from the class that in this case, as in many other applications, x and y will be replaced by other variables.

- Challenge the class to answer the question: After how many events will these two ambitious college students become millionaires? Elicit that by substituting I = 1,000,000 in the equation I = 5000n – 50000, we have 210 events. Elicit the meaning of mathematical modeling in the context of this example, i.e., we were able to answer this question without going back to the original problem but rather the equation. Use the graphing calculator to verify that the calculator gives the same answer that the hand plotted graph gave. Use the table set feature and let [TblStart] = 0, and [ΔTbl] = 5. The graph will be a very small piece of the function. Use the [Window] to show the full picture. Set [Xmin] = 0, [Xmax] = 30, [Ymin] = 0 and [Ymax] = 1,000,000. After viewing the graph, view the table and show that when n = 210, I = 1,000,000.

- Summarize with the students the steps in converting a verbal problem into a linear function.
  1. Identify the variables and use letters to represent them.
  2. Translate the relationship into a sentence.
  3. Express your sentence using variables and function notation.· Use the data points to find the slope and y-intercept of the function.
  4. Write the equation using y = mx + b form.
  5. Specify the domain based on the given information.

- Have the class convert the following verbal problem into a linear function:
  Ms. Williams is planting a garden. It costs her $200 to buy her gardening tools. Every packet of seeds she buys to plant in her garden costs her $2.50. Write an equation that expresses the total cost of the garden as a function of the number of seed packets purchased.
  For this example, a solution may be C(p) = 2.50p + 200 where 'C' is the total cost of the garden and 'p' is the number of seed packets.

- In order to introduce the idea of a step function and its graph, pose the following example:
  It costs 50 cents for the first minute of a long distance telephone call and 20 cents for each additional minute or fraction thereof. Give a graphical model for a phone call that lasts t minutes. In the discussion of this problem lead the class through the creation of a table that categorizes the cost of phone calls. Because a call lasting 2.2, 2.5, 2.7 or 2.9 minutes costs the same as a call lasting 3 minutes, this results in a graph different from many previous graphs. It is called a step function because its graph resembles a flight of steps.

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