TOPIC 1.6: SOLVING QUADRATIC EQUATIONS

PERFORMANCE OBJECTIVES

Students will be able to:

- solve quadratic equations by factoring, by completing the square and by using the quadratic formula
- decide which of these three methods is most appropriate to use
- derive the quadratic formula
- explain what is a discriminant and for what it is used

MATERIALS

Overhead projector, prepared transparencies

STRATEGIES (This lesson may require two days.)

- This lesson will allow students to choose different methods for solving quadratic equations. Cooperative groups can easily be used for this lesson since part of this lesson is a review. The groups enable students to ask each other questions to gain an understanding about the details of the solutions. The derivation of the quadratic formula is also reviewed here.

- Consider beginning the lesson by using the following set of instructions: Compare and contrast the solutions of these two quadratic equations

  (a) $x^2 = 25$ (b) $(x + 1)^2 = 25$

- From this activity elicit and discuss that the solution of (a), an incomplete quadratic equation, involves taking the square root of both sides, and that (b), a structurally similar quadratic, may also be solved by taking the square root of both sides. Emphasize that taking the square root of the right side of both equations yields ± 5.

- Have the solutions of (b) worked out on a transparency (1) by factoring, (2) by the quadratic formula, and (3) by taking the square root of both sides. Compare the methods to elicit that the solutions are the same regardless of the method.

- Challenge the class to solve the following quadratic by completing the square: $x^2 - 4x = 12$. Contrast the solution to factoring. Use a second example such as the following to illustrate the introduction of fractions when completing the square: $x^2 - 5x = 14$.

- Pose the example: $2x^2 - 12x - 7 = 0$ to solve by completing the square. Elicit that the leading coefficient of x^2 is not a perfect square and that in order to get the left side of the equation to be a perfect square, we need to divide not only the leading term by 2, but every term by 2.

- If time permits, have a transparency prepared for the derivation of the quadratic formula starting with the general quadratic form $ax^2 + bx + c = 0$. Use the review of this proof as an example of how to read mathematics, as well as, derive the formula.

- Have the class decide which method of solution is most efficient to solve each of the following examples:

  (a) $x^2 + 6x + 10 = 0$ (b) $\frac{4}{v} = \frac{v-6}{v-4}$ (c) $x^2 - 6x = -9$

  The solutions to these three problems yield

  (a) 3 + i, 3 – i (b) 2, 8 (c) 3, 3.

  In solving the first example, elicit that the part of the quadratic formula under the radical, $b^2 - 4ac$, is what determines that the roots are imaginary. Summarize that if $b^2 - 4ac < 0$ (negative), the roots are imaginary. In the third example, $b^2 - 4ac = 0$, a result that produces two roots that are the same. Summarize that if $b^2 - 4ac = 0$, the roots are numbers that are real, rational and equal to each other. In the second example, $b^2 - 4ac$ is a positive number that is a perfect square. Its square root is therefore positive and produces two different roots that are numbers that are real and rational. Elicit that if $b^2 - 4ac$ is not perfect square, it will produce two different roots that are real but contain radical expressions, hence, irrational. The expression $b^2 - 4ac$ is called a discriminant. The results of this discussion may be summarized in table form on an overhead transparency. Further elicit that if the discriminant is a perfect square, the quadratic expression of the equation factors.

- Have the class solve the following radical equation $\sqrt{2x+5} = x + 1$ and relate it to solving a quadratic equation (i. e., after the squaring of both sides step is done.) Be sure to discuss the importance of only using the positive root since the negative root is rejected here.

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