TOPIC 1.7: GRAPHING QUADRATIC FUNCTIONS

PERFORMANCE OBJECTIVES

Students will be able to:

- use graphing calculator to plot the graph of a quadratic function
- verify algebraic solutions to quadratic equations by inspecting the graph of its equation
- graph a quadratic function by finding the x, y intercepts, the vertex and the axis of symmetry
- transform and graph an equation of a parabola in the standard form $y = (x - h)^2 + k$

MATERIALS

Graphing calculator, prepared transparencies

STRATEGIES (This lesson may require at least two days.)

- Begin the lesson with the following Do Now problem:

  (a) Instruct students to graph $y = x^2 - 8x + 7$ on the graphing calculator. Inform students that they may press the following sequence of buttons on the TI-83 graphing calculator: [Y=], [X,T,θ,n], [x^2],[-], [8], [X,T,θ,n], [+], [7], [ZOOM], [6].

  (b) The following introduces the "trace." Instruct students to press: [Y=], [▼ the blue down arrow key], [◀ ◀ the blue left arrow key twice], [ENTER], [ENTER], [ENTER], [ENTER], [▶ ▶ the blue right arrow key twice], [( ] , [X,T,θ,n], [-], [4], [ )], [x^2], [-], [9], [GRAPH]. Explain how this equation is related to the previous one. (This introduces the idea of the trace "tail" when the graph is plotted. Since it will be the same graph, without it, the calculator seems to be plotting just one graph.)

  (c) Find the turning point of the parabolas.

- Once the class realizes that both of these graphs are exactly the same, elicit that they are the graph of the parabolas $y = x^2 - 8x + 7$ and $y = (x - 4)^2 - 9$ and challenge them to find the turning point of the graph by using the [2^{nd}], [TRACE] menu. Elicit that the turning point is the minimum value of the graph and that on the [CALC] menu, we enter [3: minimum] in order to find it. Further elicit that the minimum value of the parabola is somewhere between 1 and 8. When prompted for a Left Bound, enter 1, and when then prompted for a Right Bound, enter 8. When prompted to guess, enter [ENTER] and the coordinates for the turning point of the parabola are displayed (3.9999971, -9) or just (4, -9). Challenge the class to compare the coordinates of the turning point or vertex is (4, -9) and the equation $y = (x - 4)^2 - 9$ and elicit that the coordinates of the turning point is the same as the numbers in the equation of the parabola, something that would suggest a new standard form. Summarize the discussion as follows: "If the equation of the parabola is in the form of $y = (x - h)^2 + k$, then the coordinates of the turning point are (h, k). Alternatively, the standard form may be interpreted as $y - k = (x - h)^2$." To reinforce this concept, have the class algebraically simplify the second equation to show that it is the same as the first.

- Elicit that the equation for the axis of symmetry is x = $\frac{-b}{2a}$ and for the above example, the equation of the axis of symmetry is x = 4, a result consistent with the turning point being (4, -5). Have the class inspect the graph for the x and y intercepts (1, 0), (7, 0) and (0, 7). Challenge the class to find the intercepts from the equation $y = x^2 - 8x + 7$. Elicit that by letting x = 0 and substituting this value into the equation, we get y = 7, or the y intercept, (0, 7). Similarly, by setting y = 0, we have a quadratic equation whose roots are 1 and 7, giving the x-intercepts (1, 0) and (7, 0).

- The second part of this lesson can be may be started with practice on the completing the square method and the new standard form of a parabolas. Have the class find the vertices of the following parabolas algebraically by completing the square and then checking their work with the calculator: $y = x^2 + 4x - 2$ [vertex (-2, -6)], and $y = x^2 + 3x + 5$ [vertex (1.5, 2.75)]. For the example, $y = - x^2 + 6x + 2$ [vertex (3, 11)], lead the class through the procedure of rewriting the equation into $y = - (x^2 - 6x \quad) + 2$, before completing the square on the expression contained in the parenthesis.

- Graph, on the graphing calculator, $y = x^2$, $y = 2x^2$, $y = \frac{1}{2}x^2$ on the same set of axes to see how modifying the coefficient *a* in $y = ax^2$ changes the graph of the equation. Elicit informally that a larger value of the coefficient *a* results in a "skinnier" parabola. Elicit the effect of a positive coefficient between 0 and 1, as well as, the effect of a negative value of a coefficient *a* on the parabola. After doing this have the class find the turning point of $y = 2x^2 + 8x + 5$. This transforms into $y = 2(x + 2)^2 - 3$. Elicit that the graphical significance of the leading coefficient being two in this example is that the parabola will be skinnier.

- Challenge the class to name the intercepts for the equation $y = (x - 2)^2$. In this example, the parabola is tangent to the x-axis, and the intercepts are (2, 0) and (0, 4). Elicit that when the parabola is tangent to the x-axis, it has a double root of x = 2 and x = 2. Review that this is the same as learned in previous classes that when the discriminant $b^2 - 4ac = 0$, the roots are equal. Have this prepared on a transparency.

- Challenge the class to name the intercepts for the equation $y = (x - 2)^2 + 1$. With this example, the class will readily see that there are no x-intercepts and that the y-intercept is (0,5). Point out that this is the same as the discriminant $b^2 - 4ac < 0$ and the roots being imaginary. Have this prepared on a transparency.

- A third aspect of this lesson deals with linear quadratic systems. Pose the example "Where does the line y = 2x + 5 intersect the parabola $y = 8 - x^2$?" Lead the class through the algebraic solution (it may be done on a prepared transparency) and then a solution by the graphing calculator. Have them enter each equation on the [Y=] menu and [GRAPH]. To find the points of intersection using the TI-83, press [2^{nd}], [TRACE], [5:intersect]. When the calculator prompts for "First curve," move the cursor near one of the points of intersection using the blue arrow keys and then press [ENTER]. When the calculator prompts for "Second curve," simply press [ENTER], and [ENTER] again. It will display one of the two intersection points (1, 7) or (-3, -1).

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