TOPIC 1.8: QUADRATIC MODELS

PERFORMANCE OBJECTIVES

Students will be able to:

- write a quadratic equation in the form $y = ax^2 + bx + c$ given three of its values
- write a quadratic equation from data given in a verbal example
- model real life situations using quadratic functions

MATERIALS

Graphing calculator, graph paper, prepared transparencies, overhead projector

STRATEGIES (This lesson may require two days.)

- Pose the following Do Now:
  Explain if (1, 10) is a solution of the equation $y = 3x^2 - x + 2$.

- Elicit from the class that the basic concept in the do now is: A point is a solution to a quadratic equation in two variables if and only if its coordinates may be substituted for the variables of the equation and the results are true.

- Pose the converse, i.e., "How do we find a quadratic equation given three values based on the function?" For example if f(1) = 3 and f(2) = 5 and f (3) = 9, how do we derive the quadratic equation that represents f(x). If the standard form of a quadratic equation: $f(x) = ax^2 + bx +c$, challenge the class to find f(1) in terms of a, b and c. Elicit f(1) = a + b + c. Also elicit that if f(1) = 3, a + b + c = 3, Similarly, f(2) = 5 = 4a +2b + c and f(3) = 9 = 9a + 3b + c. Solving the system of equations for a, b, and c is essential to finding the quadratic equation that generates the quadratic function that generates the given values.

  $f(1) = a(1)^2 + b(1) + c = 3 \Rightarrow a + b + c = 3$ (equation I)
  $f(2)= a(2)^2 + b(2) + c = 5 \Rightarrow 4a + 2b + c = 5$ (equation II)
  $f(3)= a(3)^2 + b(3) + c = 9 \Rightarrow 9a + 3b + c = 9$ (equation III)

  Discuss with students how to solve three equations in three unknowns. In this example, in order to solve for 'a', first subtract equation I from equation II, 3a + b = 2 (equation IV) Then subtract equation 2 from equation 3, 5a + b = 4 (equation V). Subtracting equation V from equation IV results in: 2a = 2 or a = 1. To solve for 'b', substitute a = 1 in equation IV which yields b = -1. Finally, substitute a = 1 and b = -1 into either equation I, II or III to get c = 3. With a = 1 b = -1 and c = 3, we can write the quadratic equation by substituting them into the standard from of a quadratic equation to get: $\mathbf{f(x) = x^2 - x + 3}$**.** This derivation may be done on a prepared acetate ahead of the class.

- Since solving simultaneous equations is tedious and time consuming, explain how to use a graphing calculator to find the values of a, b and c for a quadratic equation, $y = ax^2 + bx + c$. On a TI-83, you use the following steps: Using the previous example, f(1) = 3, f(2) = 5, and f(3) = 9. Press the [STAT], [1], [ENTER]. (This will give you a screen showing Lists (L1, L2, L3…). In order to enter the x values: 1, 2 and 3, press [1], [ENTER], [2], [ENTER], [3]. In order to enter the y values of 3, 5, and 9, press [blue ▶], (to go to List 2) and press [3], [ENTER], [5], [ENTER], [9]. In order to calculate the quadratic function that generates this list, press [STAT], [blue ▶], [CALC], [5], [ENTER],to get a quadratic regression WITH a = 1, b = -1 and c = 3. Therefore $f(x) = x^2 – x + 3$.

- Define quadratic regression as the process of finding a quadratic function that passes through three given points. Challenge the class to find the quadratic function that passes through (1, -2), (2, 3) and (3, 10). Use the graphing calculator to find $y = x^2 + 2x – 5$.

- Pose the following example for the class: A new hat store on Flatbush Avenue sells the latest sports designer hats. The store charges $15 for one designer hat and has been selling about 500 of them per week. With the popularity of these hats among teenagers, the storeowner estimates that for every $1 price reduction, 300 more hats will be sold per week.

  (a) Write a quadratic formula R(x) that gives the total revenue received by the store in a week.
  (b) What price will maximize the total revenue?

- Challenge the students to interpret the information given above. Elicit the following: R(15) = 7500, R(14) = 11200 and R(13) = 14300 Have the class enter the data on their calculator and find a = -300, b = 5000 and c = 0, so $R(x) = -300x^2 + 5000x$. Elicit from the class that that price will correspond to the x coordinate of the turning point which is also obtained by use of the formula: $x = \frac{-b}{2a}$, therefore $x = \frac{-50000}{2(-300)} = 8.33$. Students can also find this value using the graphing calculator by entering the equation on the [Y=] , adjusting the window so that the maximum value on this parabolic model fits on the screen, and finding the maximum point using the [2nd], [CALC], [4] keys. The revenue at 8.33 is $20833.33.

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