

TOPIC 2.1: SYNTHETIC SUBSTITUTION

PERFORMANCE OBJECTIVES

Students will be able to:

- identify a polynomial function
- identify the special names given to certain polynomial degrees and give examples
- evaluate a polynomial using synthetic substitution
- determine zeros of a polynomial function

MATERIALS

Overhead projector, graphing calculator

STRATEGIES

- Students will learn to use the technique of synthetic substitution by discovering that directly substituting a value of the variable into a given polynomial $f(x)$ yields the same “last sum” value as in the synthetic substitution method. Students will also learn how to evaluate a polynomial expression.

- Start the lesson by posing the following Do Now:

$$\text{If } f(x) = 3x^4 - 7x^3 - 5x^2 + 9x + 10 \text{ and } g(x) = x(x(x(3x - 7) - 5) + 9) + 10$$

- (a) Find $f(1)$ and $g(1)$
 - (b) Find $f(2)$ and $g(2)$
 - (c) Explain what you notice about your results for (a) and (b)
- Have students go to the board and show that $f(1)$ and $g(1)$ are both equal to 10 and that $f(2)$ and $g(2)$ are both equal to 0. Challenge the class to explain why both pairs of answers are equal. From this, ask the class to simplify $g(x)$ into polynomial form by using the order of operations. This demonstration may be worked out on acetate to expedite the details of this example. The vocabulary term of quartic polynomial should be included in the discussion.
- Show the class the “synthetic substitution” method in a tabular form for evaluating $g(x)$. Explain that the arithmetic operations of evaluating $g(x)$ are the same as in this table and perhaps more convenient. Draw the connection between the following and the process of evaluating $g(x)$. Note that the “last sum” in the synthetic substitution is 10 and the same thing that was obtained by evaluating $g(1)$. This, in turn, is the same as $f(1)$, as the do-now demonstrated.

$$\begin{array}{r} 3 \quad -7 \quad -5 \quad 9 \quad 10 \quad [1 \\ \quad 3 \quad -4 \quad -9 \quad 0 \\ \hline -4 \quad -9 \quad 0 \quad 10 \end{array}$$

- The medial summary should be: When given $f(x)$, $f(a)$ may be found in one or two ways:
 - (a) by direct substitution into $f(x)$ or
 - (b) by using synthetic substitution so that the “last sum” in the bottom row is $f(a)$.
- Have the class find $f(3)$ if $f(x) = 2x^5 + 3x^4 + 10x^2 + 7x + 15$ using both direct substitution and synthetic substitution. Include in the discussion that this is a quintic polynomial. The last example requires the use of a zero for a placeholder in the top row in the synthetic substitution for the cubic term.
- Summarize the concept of synthetic substitution by posing the following example:
Find all of the zeros of the following polynomial given that one of the zeros is -1 :
 $f(x) = 2x^3 - 5x^2 - 4x + 3$. Explain that the zeros of a polynomial are the values of the polynomial that make the value of the polynomial equal to zero.

Lesson plan by Patrina Fulton