

TOPIC 2.2: SYNTHETIC DIVISION

PERFORMANCE OBJECTIVES

Students will be able to:

- divide polynomials using synthetic division
- state and apply the Remainder Theorem
- determine whether or not a divisor is a factor of the dividend as per the Factor Theorem
- factor a cubic polynomial knowing the divisor and quotient of synthetic division

MATERIALS

Overhead projector

STRATEGIES

- Students will learn that synthetic division will yield the same quotient and remainder as long division. The students will understand that synthetic division gives the quotient and remainder (if any) and the importance of when the remainder is zero.
- Start the lesson with the following Do Now:
 - (a) Find the remainder when $2x^4 - 15x^2 - 10x + 5$ is divided by $x - 3$
 - (b) If $f(x) = 2x^4 - 15x^2 - 10x + 5$, find $f(3)$ using synthetic substitution.
 - (c) What do you notice about the results in (a) and (b)?
- Elicit that the results of (a) and (b) are the same. Challenge the class to contrast the numbers under the third row of the synthetic substitution with the quotient of (a). Summarize and formally state the results of the do now as the Remainder Theorem. Reinforce the concept that $x - a$ is the divisor of $P(x)$, but "a" is the divisor of the synthetic division procedure. Formally state that synthetic substitution algorithm is called synthetic division.
- Challenge the class to answer the following problem:
Use synthetic division to find the quotient and remainder if $P(x) = x^3 + 5x^2 + 5x - 2$ is divided by $x + 2$. Have the class verify that the quotient of the preceding example is correct by multiplying the divisor by the quotient, and to that product add the remainder. Formally state the Factor Theorem: For a polynomial $P(x)$, $x - a$ is a factor if and only if $P(a) = 0$.
- Elicit that the numbers of bottom line of the synthetic division represent the coefficients of the quotient. Further elicit that by knowing this, we can factor a cubic polynomial. In particular, $x^3 + 5x^2 + 5x - 2 = (x + 2)(x^2 + 3x - 1)$. Further elicit that if the original dividend is a cubic polynomial, the numbers under the line of synthetic division will be quadratic.

- Summarize concepts of the lesson with the following problem:
Use the Factor Theorem to decide which of the following is a factor of
 $P(x) = x^4 - 3x^3 + 5x - 2$

(a) $x + 2$
(b) $x - 2$
(c) $x + 4$

When setting up the synthetic division, elicit that the x^2 term is missing and that a zero must be used in the algorithm.

Lesson plan by Andrew Song