TOPIC 2.3: GRAPHING POLYNOMIAL FUNCTIONS

PERFORMANCE OBJECTIVES:
Students will be able to:

- graph a polynomial function using a calculator
- determine an equation for a polynomial graph given its zeros
- sketch a polynomial from the equation and it roots
- determine the degree of a polynomial function given its graph
- determine the effect of double or triple roots on the graph of an equation

MATERIALS:

Overhead graphing calculator, graphing calculator, prepared transparencies

STRATEGIES: (This lesson may require two days.)

- The opening activity of this lesson shows the effect of positive or negative leading coefficients on the graph of a cubic equation. Consider starting the lesson by posing the following example:

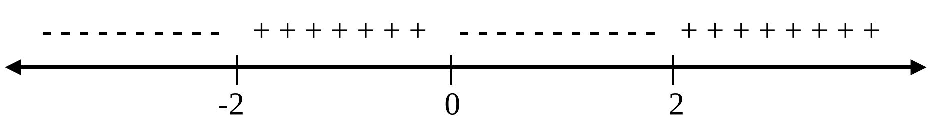
  (a) Graph the following using your graphing calculator $y = x^3 - 4x$. Copy the result on a sheet of paper.
  (b) Graph $y = -x^3 - 4x^2 - x + 6$ and copy the result on sheet of paper.
  (c) Explain basic differences and similarities in these two graphs.

- From this activity elicit and discuss that generally the graphs of $y = ax^3 + bx^2 + cx + d$ is continuous and if $a > 0$ then the curve rises from left to right and if $a < 0$ the curve falls from left to right. Further elicit that the x-intercepts (zeros) of the graph of (a) are –2, 0 and 2, and the x-intercepts (zeros) of (b) are -3, -2 and 1.

- Have the students factor the two cubic expressions of the first example. Elicit that the zeros of the graph and the x-intercepts are directly related to the factored form of the equation. For (a) the factored form is $x(x - 2)(x + 2)$ and for (b) the factored form is $(x + 3)(x + 2)(x - 1)$.

- Determine the sign graph for each cubic equation by
  (1) drawing a number line and placing points over the three zeros of the cubic equation.
  (2) + signs in the intervals where the graph produces positive values [graph above the x-axis].
  (3) – signs in the intervals where the graph produces negative values [graph below the x-axis].

For $y = x^3 - 4x$, the sign graph should look like:



The class can also do the sign graph of (b) in a similar manner.

- Challenge the class to sketch an approximation of the graph of $y = x(x + 2)(x – 1)$ without the use of the graphing calculator. After, verify the accuracy of the graph by drawing it on the calculator.

- In order to explore the effects of double roots of a cubic polynomial, ask the class to graph $(x + 1)(x – 3)^2$. Elicit that at the double root of $x = 3$, we have the graph being tangent to the x-axis, as was the case in the parabola in a previous course. To show the effects of a negative coefficient on a cubic equation with double roots, have the students graph $–x(x – 4)^2$ on their calculators.

- In order to explore the shapes of the quartic graphs, have students use the calculator to graph of $y = (x – 2)(x – 3)(x + 3)(x + 1)$. Have them identify the x intercepts and the shape. Have a prepared transparency for $y = –(x + 3)(x + 1)(x – 1)^2$, showing the effect of the leading coefficient being negative, as well as, the double root at $x = 1$.

- Finally, have the class graph a quartic equation having a triple root. Challenge them to sketch the graph of $y = (x + 2)^3(x + 4)$. Discuss the inflection point at $x = 2$.

- Summarize the lesson as follows:

    1. When sketching a cubic polynomial function, if $a > 0$ the graph rises from left to right and if $a < 0$, the graph falls from left to right.
    2. When sketching a quartic polynomial function,
        (a) if $a > 0$, the graph is a north opening “W.”
        (b) if $a < 0$, the graph is a south opening “M.”
    3. The zeros of the polynomial are the same as the x-intercepts.
    4. When the graph is above the x-axis, the function is positive, and when it is below the x-axis, the function is negative.

Lesson plan by Victoria Cato and Tracy Saltwick