TOPIC 2.4: FINDING THE MAXIMUM AND MINIMUM VALUES OF POLYNOMIAL FUNCTIONS

PERFORMANCE OBJECTIVES

Students should be able to:

- write a polynomial function given a real life situation
- graph a polynomial function
- tell whether a function will have a maximum or minimum point by looking at graph
- use a calculator to find the maximum and minimum of the function

MATERIALS

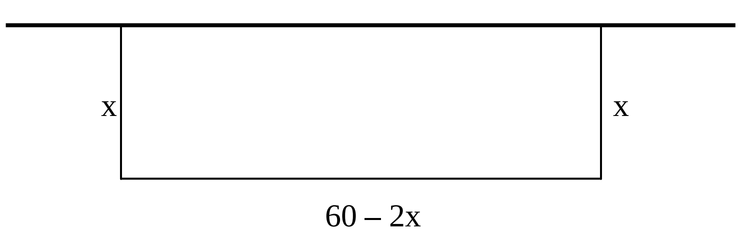
Overhead projector, graphing calculator

STRATEGIES

- Start the lesson with the following Do Now:
  "Farmer John's pigs have been running wildly around the farm. He decides to build them a regular pigpen using the barn as one side and 60m of fencing for the other three sides. Find the dimensions of the pigpen that gives the greatest area for his pigs."

  This problem should introduce the class to mathematical modeling, i.e., using an equation as a point of analysis of a situation. The concept of maximum and minimum values can be introduced and explored with the graphing calculator without derivatives.

- The class should be led through the steps needed to solve the do now. Encourage the use of
  (1) a labeled diagram.
  (2) identifying what has to be maximized or minimized.
  (3) writing an expression in terms of x for that which is being minimized.
  (4) the graphing calculator to identify the maximum or minimum point on the graph of the expression.



$A(x) = x(60 - 2x) = -2x^2 + 60x$. In many of these problems, the function that is being maximized does not fit on the screen of the graphing calculator. Have the class algebraically determine that the x intercepts of this graph are (0,0) and (30, 0).

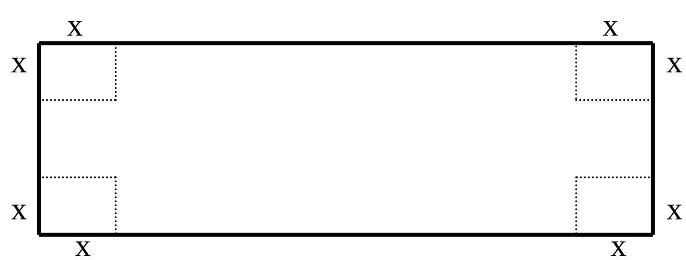
Using this information, reset the x_{min} to 0 and the x_{max} to 30 on the window menu and graph. Since this is the graph of a parabola, use the $x = \frac{-b}{2a}$ formula to locate the axis of symmetry or reason that the axis of symmetry is equidistant from the two x-intercepts on the graph. Both give the answer of the vertical line x = 30. Graph the equation again to see that the height is wrong. Adjust the y_{min} to 0 and the y_{max} to 200, 300 400 or 500 until it becomes clear that the height of the graph is correct for the window and the entire parabola is on the graph. To find the maximum of this function using the TI-83, press [2^{nd}], [TRACE], [4]. When prompted for "Left Bound," enter a value to the left of where the graph is at its maximum, for example, 10. Press [10], [ENTER]. When prompted for "Right Bound," similarly enter a value to the right of where the graph is at its maximum, for example, 20. Press [20], [ENTER]. When prompted for "Guess," press [ENTER] again and the calculator will give a value of (15.000003, 450) or (15, 450).

- At this point of the lesson, students will need to understand the concept of "Mathematical Modeling." The equation and the graph of the equation are a model for what is happening in the problem of Farmer John. The x-axis is the value of the two segments marked in the diagram as x. The y-axis is the value for the area of the rectangular pigpen at this specific x. The dimensions of the pigpen when x = 15 are 15 and 60 – 2x = 30. These dimensions produce an area of 450. Challenge the class to calculate the area of any rectangle when x = 20 and 60 – 2x = 20. Elicit that the area of this pigpen would be 400, clearly less than the area when x = 15.

- Discuss the domain by challenging the class to identify the area when x = 32. Elicit that 60 – 2x = –4, producing an area of –128. Clearly this is impossible for an area. Ask the class to identify the largest value of x so that the area "makes sense." Elicit that x > 0 is needed for the area to be a positive answer. Similarly, elicit that x < 30 is needed in order for the area to be a positive answer. Summarize the discussion by stating that the domain of the variable x is 0 < x < 30 for this particular function.

- Challenge the class with the following example:
  An open box is to be made from a 30cm by 40cm rectangular sheet of metal by cutting out identical squares of side *x* from each of the corners of the sheet and folding up the sides to form a box, as shown in the figure below.

  (a) Find the approximate value of x that maximizes the volume of this box.
  (b) Give the approximate maximum volume.



Summarize the steps of setting up a minimum - maximum problem as the teacher is doing this example with the class.

(1) Write and label a diagram with the given problem.
(2) Identify that which is being maximized and write an algebraic expression that represents this.
(3) Make sure the algebraic expression is only in terms of x.
(4) Graph the function on the graphing calculator.
(5) Find the maximum point of the graph.

The x of this point is the size of the square being cut off and the y is the volume of the box created by cutting the x square off and folding the sides. For this example, the function V(x) is $x(30 - 2x)(40 - 2x)$. The largest x value is 15 because anything larger would have the corner exceed the dimension of the original rectangle, so set the domain for $0 < x < 15$. The height of the curve should be set at around 3500 so that the entire curve can be seen. The maximum point on the graph is (5.657, 3032.3). In other words, cutting off a square whose side is 5.657 produces a solid whose volume is 3032.3.

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