TOPIC 2.5: USING TECHNOLOGY TO SOLVE A CUBIC EQUATION

PERFORMANCE OBJECTIVES

Students will be able to:

- identify features on the graphing calculator that can aid them in finding the roots of polynomial equations
- approximate the real roots of a polynomial equation using the Location Principle
- explain why the Location Principle can not always be used to find all real roots in a polynomial equation
- determine appropriate window settings when graphing polynomial equations
- use the graphing calculator's features to approximate the real roots of a polynomial equation

MATERIALS

Graphing calculator, overhead projector, overhead graphing calculator

STRATEGIES

- Students will learn to use the features on the graphing calculator that will help them find the real roots to a polynomial equation. On the TI-83, this will be done by using the TABLE menu, the ZOOM FIT feature and the 2^{nd} CALC ZERO feature.

- Start the lesson by assigning the following Do Now example to the class:

  (a) What are the roots of the cubic function: $f(x) = (x + 1)(x – 1)(x – 2.5)$?
  (b) Perform a sign analysis by testing one value of x from each of the intervals determined by the zeros.
  (c) What do you notice about the y-values of the function each time the function crosses a zero of the function?
  (d) Sketch the function.

- The do now reviews with the class that the sign of f(x) changes whenever the function crosses the x-axis. It leads to the Location Principle that states if f(x) is a polynomial with real coefficients and *a* and *b* are real numbers such that f(*a*) and f(*b*) have opposite signs, then between *a* and *b* there is at least one real root *r* of the equation f(x) = 0. Summarize the discussion by writing the Location Principle on the board.

- Have students type the equation on the y = menu of the graphing calculator. Then, use the TABLE feature ([2^{nd}], [GRAPH]) to obtain a table of values for x and f(x). Students should recognize that whenever the value of $f(x) = 0$, x is a root of the equation. If not all the roots are shown using the table, look for changes in sign for values of f(x). A change in sign indicates that a root is in the interval. Explore with the class that the TBLSET ([2nd], [WINDOW]) feature can be used to adjust the list of values of x by changing the Δx, the spacing between the x values on the table.

- Make students aware that the Location Principle does not work in cases when the function has a double root. Have students type in the following equation into the calculator on the [y=] menu: $(x + 2.4)(x + 2.4)(x + 1.2)$. Challenge the students to find the roots using the Location Principle. Elicit that there is a double root at $x = 2.4$, and consequently, no sign change on either side of 2.4.

- Continue the lesson by asking students to do the following:
  (a) Graph the following equation on the graphing calculator: $y = x^3 + 10x^2 – 11x$
  (b) From the appearance of the graph using the standard window, how many roots do there appear to be?
  (c) How do you know that your answer from part (b) can not be right?
  (d) Find the other real root of this cubic equation.

- Though the graph of the equation above seems to be a parabola (quadratic equation) when graphed on the calculator because of its shape and the number of x-intercepts, students should be able to realize that this is not possible because the equation is a cubic. Part (d) reinforces the fact that the graph can not be complete since there are three real roots in this equation. Introduce students to the WINDOWS key, as well as ZOOM key. Lead the students through a discovery of the ZOOM FIT Key and elicit that by using it, the calculator will show most of the graph in the window.

- Show students how roots can be found using a TI-83. Have the students graph the following: $f(x) = x^3 - x^2 - 5x + 2$. Adjust the window if necessary. Then use the CALC feature to find the zeros. Review the procedure with the class: [2nd] [TRACE], [2], [ENTER], [left bound], [ENTER], [right bound], [ENTER], [guess], ENTER. Give more examples if practice is needed.

- If the number of roots does not equal the degree of the equation, what are some possible explanations? Elicit that either parts of the graph are missing or that the window should be adjusted because the function contains a double root, or perhaps, some are imaginary roots.

- Summarize the main points of the lesson by having students find the roots of the following: $f(x) = x^3 – 3x^2 + 4$. This is an example of a function with double roots.

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