TOPIC 2.6: SOLVING POLYNOMIAL EQUATIONS BY FACTORING

PERFORMANCE OBJECTIVES

Students will be able to:

- solve a given polynomial equation by group factoring
- recognize and factor a higher degree bi-quadratic polynomial equation
- state and apply the Rational Root Theorem
- use synthetic substitution and a graphing calculator to determine whether the possible roots found by using the Rational Root Theorem are roots.
- determine the most appropriate method to use to solve a polynomial equation

MATERIALS

Overhead projector, graphing calculator

STRATEGIES (This lesson may require two days.)

- Use the following Do Now to get the lesson started:
  Write an explanation of why $7x - 5$ can not be a factor of $12x^2 - 8x - 15$.

- Have students suggest the possible factors of the above trinomial by eliciting its possible factors. $7x - 5$ can not be one of them because $7x$ is not a factor of $12x^2$, even though 5 is a factor of 15. This activity illustrates the basis of the Rational Root Theorem. Have this theorem written out on a transparency for the class: "Let P(x) be a polynomial of degree n with integral coefficients and a non zero constant term:
  $P(x) = a_n x^n = a_{n-1}x^{n-1} + \ldots + a_0$, where $a0 \neq 0$. If one of the roots of the equation $P(x) = 0$ is $x = \frac{p}{q}$, where p and q are nonzero integers with no common factor other than 1, then p must be a factor of a_0 and q must be factor of a_n.
  Illustrate this with the example: $(3x - 16)$ is a factor of $3x^3 - 16x^2 - 12x + 64 = 0$ and $x = \frac{16}{3}$ is a root. 3 divides 3 and 16 divides 64.

- Use this preceding example of $3x^3 - 16x^2 - 12x + 64 = 0$ to have the class list all of its possible rational roots. Elicit ±1, ±2, ±4, ±8, ±16, ±32, ±64, $\pm\frac{1}{3}$, $\pm\frac{2}{3}$, $\pm\frac{4}{3}$, $\pm\frac{8}{3}$, $\pm\frac{16}{3}$, $\pm\frac{32}{3}$, and $\pm\frac{64}{3}$ for a total of 28 possible rational roots. Have the class graph $y = 3x^3 - 16x^2 - 12x + 64 = 0$ and look for the zeros at x = -2, 2, and $\frac{16}{3}$.

Verify that these are, in fact, the zeros by either using the table feature on the calculator and/or by using synthetic substitution on each of the numbers x = -2, 2, and $\frac{16}{3}$.

- Pose the following example to the class: $12x^4 - 8x^2 - 15 = 0$ to elicit the method applied to factor a higher degree **bi-quadratic** polynomial equation. Have the students compare this problem to the do now and share what they discover. Define a bi-quadratic as a quartic expression lacking the cubic and linear term. When attempting to factor it, we use the same approach that we use to factor a quadratic, i.e., $(6x^2 \quad)(2x^2 \quad)$, and use FOIL multiplication to verify. For this example, $(6x^2 - 5)(2x^2 + 3) = 0$ produces solutions of $6x^2 - 5 = 0$ or $2x^2 + 3 = 0$, and consequently $x = \pm\sqrt{\frac{5}{6}}$ or $x = \pm\, i\sqrt{\frac{3}{2}}$.

- Introduce the following example to start the factoring method of group factoring: Factor the expression in the first set of parenthesis of $(x^3 - 2x^2) - (x - 2)$ By doing this we have $x^2(x - 2) - (x - 2)$. Outline the common factor $(x - 2)$ to make the GCF more obvious to the class: x^2**(x – 2) – (x – 2)**. In factoring out this common factor we have $(x - 2)(x^2 - 1) = (x - 2)(x - 1)(x + 1)$. Challenge the class with another similar example:

  Solve: $x^3 + 5x^2 - 4x - 20 = 0$. In this example group factor the first two terms and group factor the last two terms to get: x^2**(x + 5)** – 4**(x + 5)** = 0. Factor the (x + 5) to get $(x + 5)(x^2 - 4) = 0$ and finally $(x + 5)(x + 2)(x - 2) = 0$. So x = -5, -2, and 2. The teacher may wish to consider practice with group factoring with either another example or in the assigned homework.

- Summarize the three methods for solving quartic equations in this lesson by having students first identify the most appropriate methods they would adopt to solve the following problems:

  (a) $3x^4 + 2x^2 - 4 = 0$ (bi-quadratic)
  (b) $x^3 + 6x^2 - 4x - 24 = 0$ (group factoring)
  (c) $x^3 + 2x^2 - 6x = 12$ (group factoring)
  (d) $3x^3 + 8x^2 = 9x - 2$ (rational root theorem)
  (e) (e) $x^6 - 3x^3 + 6 = 0$ (bi-quadratic)

Lesson plan by Wan-tseng Chen and Tricia Weisberg