TOPIC 3.1: LINEAR INEQUALITIES - ABSOLUTE VALUE

PERFORMANCE OBJECTIVES

Students will be able to:

- define and solve a linear inequality and an absolute value inequality
- identify the solutions of $|x| < a$ and $|x| \leq a$ as "and" statements
- identify the solutions of $|x| > a$ and $|x| \geq a$ as "or" statements
- know that the direction of shading for an inequality graph will match the inequality sign

MATERIALS

Overhead projector, graph paper

STRATEGIES

- As a Do Now ask the students to find all the real numbers that satisfies each of the following problems.
  (a) $|x| < 3$
  (b) $|x - 1| < 3$
  (c) $|2x - 1| < 3$

- Have students go the board and solve each equation. Many students will think that the solution to (a) is -2, -1, 0, 1 and 2, mistakenly thinking that the domain is integers. Review the solution using the real numbers as the domain. Use a number line to graph the solution of (a) and challenge them to use a compound inequality to express the answer. Show the class that the absolute value inequality can be rewritten into its corresponding compound inequality for each of the do now problems. Then proceed to solve them in the manner described below:

  (a) $-3 < x < 3$
  (b) $-3 < x-1 < 3$ and adding 1 to each of the three elements of the inequality yields $-2 < x < 4$. This is otherwise stated as $x > -2$ and $x < 4$.
  (c) $-3 < 2x -1< 3$ and adding 1 to each of the three elements of the inequality yields - $-2 < 2x < 4$, and dividing each of the three elements by 2 yields $-\frac{2}{3} < x < 2$. This is otherwise stated as $x > -\frac{2}{3}$ and $x < 2$. Use a number line to graph each of the preceding examples. Review that because the equation uses < and not $\leq$ that the graph of the solutions must exclude the endpoint of the indicated interval and that this can be done by having an open circle by the endpoint on the line.

- Rewrite the three inequalities of the do now using the > instead of <. Have the students solve and contrast them with the previous examples. Graph the solutions on a number line. The solution of $|x| > 3$ is $x < -3$ **or** $x > 3$.

  Challenge the class to identify the numbers that are less than -3 **and** at the same time greater than 3 and elicit that there are **no** such numbers. Contrast the solution to a ">" or a "≥" inequality to the "<" or "≤" inequality. The solution of a "greater than" inequality produce two intervals connected with an ***or*** unlike the solution of a "less than" inequality which produces one interval which may be written as two inequalities connected with an ***and*** or a single compound inequality. In addition, the graph of an ">" or "≥" inequality has two separate shaded intervals on the number line graph and not just one interval as was the case with "<" or "≤".

- Summarize the solution of an absolute value inequality by the following: To solve an absolute value linear inequality:

  (1) Rewrite the absolute value inequality into a compound inequality using the model:
  $|x| < a \longleftrightarrow -a < x < a$.
  (2) Use the exact algebraic expression enclosed within the absolute value symbols when applying (1).
  (3) Solve the resulting compound inequality for *x*, i.e., make *x* the only expression between the "<" symbols. Express the solution as a compound inequality.

- If the absolute value inequality is in the form of $|x| > a$,

  (1) Rewrite the absolute value inequality into two inequalities using the model:
  (2) $|x| > a \longleftrightarrow x > a$ or $x < -a$.
  (3) Solve each inequality and express the solution as a pair of intervals.

- Pose the following examples to give a geometric interpretation to $|x - a| < b$:
  Solve for x: $|x - 2| < 4$. After the class solves this to get $-2 < x < 6$, ask the class to name the midpoint of the interval. Elicit that it is 2. Challenge the class to identify what *4* has to do with the inequality that this graph represents. Summarize the discussion by stating that the solution of $|x - 2| < 4$ represents all the points less than 4 units from 2. Generalize this to: $|x - a| < b$ represents all points less than *b* units from *a* on a number line. $|x - a| \leq b$ is the set of all points less than or equal to *b* units from *a*. $|x - a| > b$ as all points greater than *b* units from *a*. Finally, $|x - a| \geq b$ is the set of all points greater or equal to *b* units from *a*.

- Pose one example where a direction change in the inequality is required.
  For example, pose $|2 - x| \leq 4$ to the class to assess student's understanding.

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