<u>TOPIC 3.2: POLYNOMIAL INEQUALITIES IN ONE VARIABLE</u>

<u>PERFORMANCE OBJECTIVES</u>
Students will be able to:

- make a sign graph of a linear or quadratic inequality in one variable given its algebraic form
- make a sign graph of a linear or quadratic inequality given its graph
- solve polynomial inequalities in one variable

<u>MATERIALS</u>

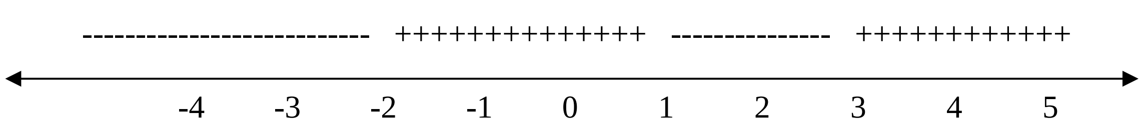
Graphing calculator, overhead projector

<u>STRATEGIES</u>
- To begin the lesson pose the following problem:

  (a) Graph $y = x^3 - 2x^2 - 5x + 6$ using your graphing calculator. (if the class does not yet know how to use one), state the Do Now as follows: Using the TI-83 graphing calculator, press the following set of keystrokes: [Y=], [X,T,θ,n], [^], [3], [–], [2], [X,T,θ,n], [X^2], [-], [5], [X,T,θ,n], [+], [6], [GRAPH]
  (b) Find the zeros from the graph.
  (c) From the graph, name the intervals where the graph is above the x-axis. Then, name the intervals where the graph is below the x-axis.

- Discuss with the class the notation for compound inequalities used in this lesson. They should write intervals as "*a* < x < *b*" or "x < c or x > d."

- Point out that, where the graph is above the x-axis, corresponds to the values such that $x^3 - 2x^2 - 5x + 6 > 0$ and where the graph is below the x-axis corresponds to the values such that $x^3 - 2x^2 - 5x + 6 < 0$. Establish that in this example the zeros determine the beginning and end points of the intervals that we have found. Take the students through the process of creating a sign graph. First have the students draw an x-axis and label the zeros with points. In the intervals where the graph is above the x-axis, indicate it with "+" symbols above the number line. In the intervals where the graph is below the x-axis, indicate it with "-" symbols. The following represents this diagram.



- From the above sign graph have the students solve this inequality: $x^3 - 2x^2 - 5x + 6 \geq 0$ by inspecting the above graph and selecting all the intervals where the + symbols are located.

For this example, the solution is $-2 \le x \le 1$ and $x \ge 3$. Notice that we use $\ge$ and $\le$ rather than $>$ and $<$ because, in this case, the zeros are included in the solution.

- Discuss with the class the fact that you can solve inequalities by using the sign graph. After marking the x-axis with the zeros you can substitute into the equation one value from within each interval in order to determine whether the function is positive or negative (i.e. above or below the x-axis). Review with the class that the results of this method are identical to the algebraic method of factoring the quadratic expression learned in previous courses.

- Pose the following to the class to motivate other points that need to be marked on the x-axis in addition to the zeros: Sketch an x-axis sign graph to indicate for what values of x the function is positive and for what values of x the function is negative: $y = \frac{1}{(x-2)(x+1)}$.

  Sketch the graph on your graphing calculator and use it to solve the inequality:
  $\frac{1}{(x-2)(x+1)} \le 0$.

- When entering the equation press the following sequence of keystrokes: [Y=], [( ], [1], [ )], [÷], [( ], [( ], [X,T,θ,n], [-] , [2], [ )], [( ], [X,T,θ,n], [+], [1], [ )], [ )], [GRAPH]. Elicit that this function has no zeros. However, it does have points where it stops being negative and starts becoming positive, and vice versa. To include this on a sign graph, elicit that the points where the function is undefined must be marked on the number line in addition to any zeros. In this case, at x = -1 and x = 2, the solution to the above inequality is $-1 \le x < 2$.

- Summarize the lesson as follows: In order to solve a polynomial inequality:

  1. determine the zeros of the polynomial.
  2. determine any points where the function is undefined.
  3. draw an x-axis sign graph and mark these points on it.
  4. test a value from within each interval in order to determine if the function is positive or negative.
  5. mark these intervals on the x-axis.
  6. list the intervals that solve this inequality. (If you are using a graphing calculator you can find the zeros and any undefined points using the graph. You can also see the intervals on the graph.)

- Have the class solve the inequality $(x^2 - 1)(x - 4)^2 \ge 0$ by using a sign graph and verify its results by using the graphing calculator.

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