TOPIC 3.4: LINEAR PROGRAMMING

PERFORMANCE OBJECTIVES

Students will be able to:

- define linear programming
- translate a verbal problem into a set of inequalities
- locate coordinates of vertices of the polygonal region formed by these inequalities
- explain why optimal points occur at the vertices of the polygonal region
- solve certain applied problems using linear programming

MATERIALS

Overhead projector, graphing calculator

STRATEGIES (This lesson may require 2 days.)

The main idea here is that, given a multiple set of linear inequality constraints, the optimum choice will be located at one of the vertices of the graphed polygonal region formed by the conditions of the problems. Graphing will allow the students to see where such vertices are located. Graphing calculators complement this lesson, as they are a quick way to see multiple constraints on a graph. They also allow a quick way to find the vertices of the polygonal region formed by graphs of the linear inequalities.

- Pose the following Do Now:

  (a) On graph paper, draw the region in the first quadrant bounded by the x-axis, y-axis, and the inequalities, $x + 2y \leq 12$, and $x + y \leq 8$.
  (b) Find the coordinates of the vertices of the polygonal region formed.
  (c) Let $P(x,y)$ be a two variable function defined such that $P(x,y) = 3x + 4y$. Find $P(x,y)$ of each of these 4 vertex points.
  (d) Find $P(x,y)$ of two points in the interior region of the polygon and compare them to the values you found for the vertex points in (b).

- Elicit from the class that the four corners of the polygon formed are (0,6), (4,4) (8,0), and (0,0) and that their respective values of P are 24, 28, 24, and 0. In addition, challenge the class to conclude after experimentation, that the value of $P(x,y)$ for any interior point is a value between 0 and 28.

- Discuss applications of linear programming. Explain how these could be constraints on manufacturing two products and the P-function expresses the profit from a particular factory. This should provide a motivation for the next problem. Often the problems will simply give a chart and ask students to find the equations of the constraints.

	Console TVs	Portable TVs	Total hours available per day
Machine A	1 hour	3 hour	18
Machine B	1 hour	1 hour	8
Machine C	3 hour	1 hour	18
Profit per TV	$70	$40	

Pose to the class, "If x represents the number of console TV's and y represents the number of portable TV's, what would be an algebraic expression, in terms of x and y, that represents the number of hours that machine A would need to manufacture x console TV's and y portable TV's?" Elicit the expression, $x + 3y$ hours. Since the constraint is that Machine A can only work 18 hours per day, the inequality that would express this would be $x + 3y \leq 18$. Similarly, for Machine B, $x + y \leq 8$ hours and for Machine C, $3x + y \leq 18$ hours. In addition, remember to elicit that both $x \geq 0$ and $y \geq 0$.

- Challenge the class to find the coordinates of the vertices of the polygonal region formed by these five inequalities above. This can be done by hand or by graphing calculator. When answering the problem by using a graphing calculator, the three inequalities can be entered as: [$Y_1 \leq (18 - x) / (3)$], [$Y_2 \leq 8 - x$] and [$Y_3 \leq 18 - 3x$]. (It is suggested that the shading feature not be used since it produces a graph that is difficult to read.) The vertex points can be read from the graph or located by using the "intersect" feature on the TI-83 graphing calculator by pressing the [2^{nd}], [TRACE], [5] feature. In this problem, three points of intersection need to be found using two lines at a time. The vertex points are *(3, 5), (5, 3), (0, 6),* and *(6, 0).* Elicit that the intersection of Y_1 and Y_3, (4.5, 4.5) is irrelevant to the solution of the problem because it is not a vertex of the polygon formed by the constraint equations.

- Challenge the class to identify the profit function P (x, y). In this case the profit made on x console TV's and y portable TV's can be expressed as 70x + 40y. Using this expression, elicit that the maximum profit will occur at a vertex point of the region. Have the class calculate the profit at each of the above vertex points as $*410, $470, $240, and $420*. Of these, have the class state that the most profitable combination is at the point (5, 3); the profit is $470. In addition, establish that any point in the interior region produces a profit less than $470 but greater than $240.

- As a medial summary, write the steps for the class needed to solve a linear programming example:

  (1) Organize the constraints into a chart.
  (2) Write constraints as algebraic inequalities.
  (3) Graph these inequality constraints.
  (4) Find each vertex point of the polygonal region formed by these linear inequalities.
  (5) Write an expression in terms of x and y for that which is to maximized or minimized.
  (6) Find the optimal vertex point by substituting each one into the two variable function.

- Offer some discussion of the importance of linear programming and dealing with multiple constraints. There are many examples that can be discussed, primarily having to do with manufacturing, business, and economics. One example comes from the power industry. A particular power plant purchases natural gas from a number of suppliers and has different stipulations in long-term contracts from each supplier. Some contracts require a fixed payment regardless of how much gas is purchased plus a per unit charge. Other contracts have a fixed payment only if some gas is purchased in a given month. Others stipulate one price for the first units purchased and a different price for additional units. The firm employs a linear programming model to figure out which contracts to employ and how much to buy from each in a given month, based on their expected demand for natural gas.

- Pose another example to summarize the procedure: In a shirt manufacturing plant, a short-sleeved shirt requires 30 minutes of labor, a long-sleeved shirt requires 45 minutes of labor, and 240 hours of labor are available per day. The maximum number of shirts that can be packaged in a day is 400, so no more than 400 shirts should be produced. Representing the number of short sleeves shirts as x and long-sleeved shirts as, y write an inequality to allocate the hours available to work in a given day. Elicit $\frac{1}{2}x + \frac{3}{4}y \leq 240$. Then, have the class write an inequality demonstrating that no more than 400 shirts can be produced in a day. Elicit $x + y \leq 400$.* Have the class graph these two inequalities considering the constraint that $x \geq 0$ and $y \geq 0$ and identify the vertex points. They are (0, 0) (0, 320), (240, 160), and (400, 0). If the profits on a short-sleeved shirt and a long-sleeved shirt are \$11 and \$16 respectively, find the maximum possible daily profit. Elicit that $P(x,y) = 11x + 16y$. Explain why the factory manager might decide to produce different numbers of shirts than the ones giving maximum profit. Elicit from the class that the three vertex points and their respective profit levels are \$0.00, \$5120, \$5200, and \$4400. Lead a discussion about the need of the purchaser and the possible inability to find a purchaser of out-of-season shirts.

*Technology Note: The two equations to be entered on the Y= menu are $y = \frac{(960 - 2x)}{3}$ and $y = 400 - x$. Note the need to rescale the calculator to see the graph. Set the window menu as $X_{min} = 0$, $X_{max} = 500$, $X_{scl} = 100$, $Y_{min} = 0$, $Y_{max} = 500$, and $Y_{scl} = 100$. The intersection of the two lines can be found using the [2nd], [TRACE], [5] feature.

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