TOPIC 4.1: FUNCTIONS

PERFORMANCE OBJECTIVES

Students will be able to:

- identify a function given various methods of representation
- determine the domain, range, and zeros of a function
- write the domain and range in compound inequality form
- determine whether a set of ordered pairs represented in any form is a function using the vertical line test
- define and identify the independent and dependent variables of a function
- identify and sketch a pinhole function in the form of y = $\frac{(\mathbf{x}-\mathbf{a})\cdot\mathbf{f}(\mathbf{x})}{(\mathbf{x}-\mathbf{a})}$
- find the coordinates of the pinhole in a pinhole function
- explain what the indeterminate form $\frac{0}{0}$ means and how it is related to pinhole functions
- explain how to differentiate between graphs containing pinholes and those with vertical asymptotes

MATERIALS

Graphing calculator

STRATEGIES

- Use the following Do Now to get this lesson started:
  (a) Sketch each of the graphs below on a different set of axes: (1) $y = 4 - x^2$ (2) $x = y^2$
  (b) If possible, find the domain and range for each of the graphs.
  (c) Which of these two graphs fails the vertical line test? What information does the vertical line test give you?

- Have the students sketch the graphs of the above equations and discuss with the class possible domains and ranges for these relations/functions. Further, review the possible zeros of the equations. Lead the discussion towards the "do now" (part 3) to determine what makes a relation a function. Discuss how to recognize a function by looking at the graphs. At this point, elicit that the vertical line test is a geometric interpretation of the definition: for any "x", there is one and only one "y". Further, ask the students how they can decide whether or not the above equations are functions just by looking at them without the graphs. End the discussion by formally defining a function on the board including the vocabulary of domain and range.

- Explore the graphs of some new functions such as those with limited domains. In order to do this, pose the following example: Use the graphing calculator to graph the following. Determine whether or not the following are functions. Explain why or why not. Find the domain and range for each.

  (a) $y = (4 - x^2)$ for $x \geq -2$ (b) $y = \frac{x^2 - 9}{x + 3}$

In example (a), it is necessary to entirely enclose $4 - x^2$ in parenthesis on the TI-83 before restricting the domain of $x \geq -2$. After entering $y = (4 - x^2)$, the restricted domain may be entered by pressing the following keystrokes on the TI-83: [( ], [X,T,θ,n], [2nd], [MATH], [4], [(–)], [2], [ )]. The result with be a part of the parabola, $y = 4 - x^2$, drawn only from $x \geq -2$. Elicit that the domain can be determined from looking at the graph and locating the "western-most point" as (-2, 0) and realizing that there is no "eastern-most point" on the graph since it continues indefinitely to the right. Consider using the template for domain: "western-most x value" ≤ x ≤ "eastern-most x value." In this case, where there is no eastern-most point, we use the symbol ∞. The domain is $-2 \leq x < \infty$. Similarly, the range can be determined by realizing that there is no "southern-most point" and that (0, 4) is the "northern-most point." Use "southern-most y value" ≤ y ≤ "northern-most y value" as the template for range. In this case where there is no southern-most point; we use the symbol, $-\infty$. The domain is $-\infty < y \leq 4$. In (b), we have a pinhole function. Have the class graph and notice that the graph seems to be the linear function $y = x - 3$.

Examine the table and notice that the y value at x = -3 is "ERROR." Elicit that when x = -3 is substituted into the function $y = \frac{x^2 - 9}{x + 3}$, we have the expression $\frac{0}{0}$. Explain that this could either be 1 (using the rule that any number divided by itself is 1), 0 (using the rule that if the numerator of a fraction is 0, the value of the fraction is 0), or undefined (using the rule that if the denominator of a fraction is 0, the value of the fraction is undefined). We call this mathematical condition "indeterminate" because we can not determine which of the three above choices it should be. Elicit that the graph is really the line $y = x - 3$, with a point deleted at x = -3. From the table, determine the pinhole deleted is the point (-3, -6). Elicit that the algebraic method of determining the pinhole is by factoring the numerator, canceling the factor x + 3 and evaluating $y = x - 3$ with x = -3. Summarize the discussion by eliciting that the domain is all real numbers except –3 and the range is all real numbers except -6.

- More practice problems that include domain, range and zeros of a function may be needed. Pose the following: (1) Use the graphing calculator to graph each function. Find the domain, range, and zeros of each of the following functions: (a) $f(x) = \frac{\mathbf{x}^3 - \mathbf{x}^2}{\mathbf{x} - 1}$ (parabola $y = x^2$ with a pinhole at (1,1)) (b) $y = \frac{4x}{x^2 + 1}$ (a new graph with no points deleted in the domain, because the denominator is always positive) (c) $y = \frac{3}{\mathbf{x} - 2}$ (a rectangular hyperbola having a vertical asymptote at x = 2) (d) $y = \frac{4x}{x^2 - 1}$ (a new graph with two vertical asymptotes at x = -1 and x = 1). Elicit that these last two examples may look like graphs with pinholes, but in these cases, no factor cancels in the numerator and denominator. (e) $y = \pm\sqrt{x + 3}$ (top and bottom halves of an east facing parabola) and (f) $y = \pm\sqrt{-\mathbf{x}}$ (top and bottom halves of a west facing parabola.

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