TOPIC 4.2: OPERATIONS ON FUNCTIONS

PERFORMANCE OBJECTIVES

Students will be able to:

- perform various operations on functions
- compose various functions
- determine the domain of the resulting functions

MATERIALS

Graphing calculator, overhead projector

STRATEGIES

- Start the lesson with the following do now problem:
  (a) If $f(x) = x^2 + x$ and $g(x) = x + 1$, find f(2) and g(2).
  (b) Multiply the result of f(2) by the result of g(2).
  (c) If $h(x) = x^3 + 2x^2 + x$, find h(2).
  (d) What do you notice about the results found in (b) and (c)?

- Elicit that the results of (b) and (c) are the same. Multiplying f(x) and g(x) will give you h(x). Thus $(f \bullet g)(x) = f(x) \bullet g(x)$. Summarize that the same applies for addition, subtraction and division as follows:

  $(f + g)(x) = f(x) + g(x)$; $(f - g)(x) = f(x) – g(x)$, $(f/g)(x) = f(x)/g(x)$, $g(x) \neq 0$.

  Using f(x) and g(x) from the preceding example, compute (a) $(f + g)(x)$, (b) $(f – g)(x)$ and (c) $(g/f)(x)$. Identify the domain of each. From this example, define the sum of two functions, the difference of two functions and the quotient of two functions.

- In order to explore the problem of performing operations with certain kinds of functions that have limited domains, pose the following: Let $f(x) = \sqrt{\mathbf{x} - 3}$ and $g(x) = x$. Use your graphing calculator to find $(f + g)(x)$. Determine that the domain is the same as f(x) and summarize that if the domain of either f or g is restricted, then the domain of f + g, f – g, f • g and f/g are restricted in the same way. Further, let $h(x) = \sqrt{-\mathbf{x} + 5}$ and compute the graph f(x) + h(x). Examine that the domain is doubly restricted by both f and h. The graph only exists in the domain $3 \leq x \leq 5$.

- Pose the following problem:
  Let $f(x) = x^2 + x$ and $g(x) = x + 1$
  (a) Find f (3).
  (b) Find g (f (3)).
  (c) Find f (g (3)).

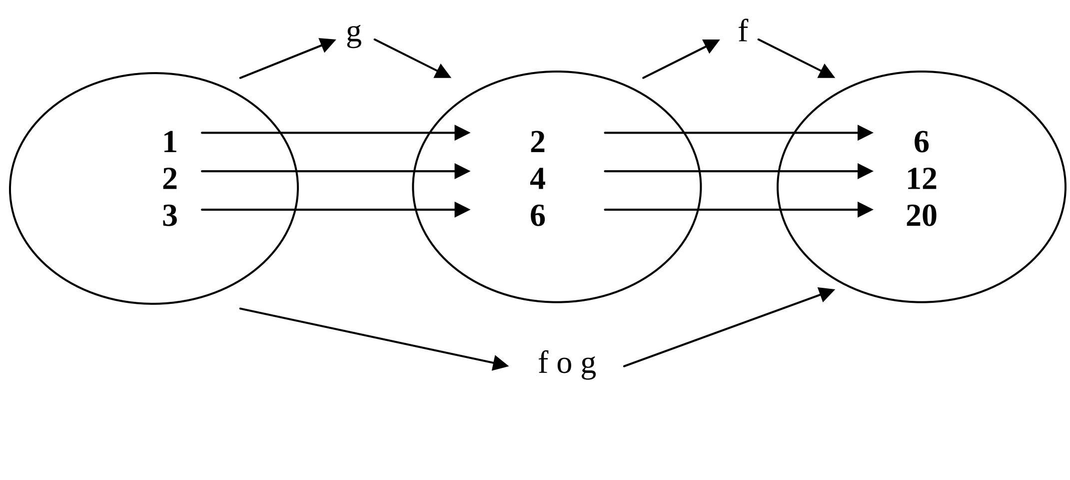
Review the procedures for forming the composition of functions with the class using this example. The composition above can be noted (g o f)(x) or "g of f". Using this problem, establish that composition is not commutative.

- In order to cover the concept of relating how the domains of f(x) and g(x) are related to the domain of (f o g)(x) and (g o f)(x), pose the following example
  Let $f(x) = x^4 - 3x^2$ and $g(x) = \sqrt{x-2}$

  (a) Find the domain of f(x) and g(x).
  (b) Find (f o g)(x).

  Find the domain of the (f o g)(x) by graphing it on your calculator.
  Challenge the class to identify the domains of f(x), g(x) and (f o g)(x). Use the graphing calculator to graph $(f \circ g)(x) = (\sqrt{(x-2)})^4 - 3(\sqrt{(x-2)})^2$ and discover that the domain is all real numbers $x \geq 2$ because of the radical from the function g. If this expression were to be algebraically simplified to $y = x^2 - 7x + 10$, the values less than 2 for x would need to be eliminated from the domain. Point out that in determining the domain of a composition, it is necessary to use the original and not simplified version of (f o g)(x). Challenge the class to identify how to determine the domain of the composition of two functions.

- If time permits, summarize the discussion by posing the following mapping diagram to help students visualize the composition of two functions. Use the example above where $f(x) = x^2 + x$ and $g(x) = x + 1$ to model the composition (f o g)(x). Point out that x is in the domain of g and g(x) is in the domain of f. The values of 1, 2, and 3 are mapped into 2, 3, and 4 under g and then mapped into 6, 12, and 20 under f. The function $(f \circ g)(x) = x^2 + 3x + 2$, when simplified, would have mapped 1, 2, and 3 into 6, 12, and 20 directly.



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