TOPIC 4.3: REFLECTING GRAPHS; SYMMETRY

PERFORMANCE OBJECTIVES

Students will be able to:

- reflect a graph, y = f(x), in the x-axis, y-axis, and y = x
- reflect a graph, y = f(x), in the origin
- use symmetry to sketch graphs
- determine whether a function is even or odd
- sketch y = -f(x), y = f(-x) or y = $|f(x)|$ when given y = f(x)

MATERIALS

Overhead projector, graphing calculator, prepared acetate transparencies

STRATEGIES (This lesson may require two days.)

- As a Do Now pose the following example to the class to start the lesson:
  On two separate sets of axes, graph $y = x^2 - 4$ and $y = |x^2 - 4|$. How are these graphs related to each other? Elicit from the class that once we know what $y = x^2 - 4$ looks like, $y = |x^2 - 4|$ is almost the same graph except the part of the graph that was below the x-axis is reflected in the x-axis. The part of the graph that was above the x-axis remains the same. Summarize that the graph of y = $|f(x)|$ is the same as y = f(x) whenever the graph is above the x-axis [f(x) > 0] and reflects whatever part of y = f(x) is located below the x-axis [f(x) < 0] in the x-axis. Elicit that y = $|f(x)|$ is never below the x-axis. Symbolically,
  $$|f(x)| = \begin{cases} f(x), & f(x) \geq 0 \\ -f(x), & f(x) < 0 \end{cases}$$

- Pose the following example to the class: Graph each pair of equations on a single set of axes.
  (a) Graph $y = x^2$ and $y = -x^2$.
  (b) Graph $y = x^3 + 2x^2$ and $y = -(x^3 + 2x^2)$.
  Have the class graph both pairs of functions and elicit how the pairs of graphs are related. Summarize that the graph of y = - f(x) is the reflection of y = f(x) in the x-axis and that if the point (x, y) is on y = f(x), then (x,-y) is on y = -f(x).

- Pose the following exploration to elicit the reflection of a graph in the y-axis:
  (c) Graph $y = 2^x$ and 2^{-x} on the same graph using your graphing calculator
  (d) Graph $y = x^3 - x^2$ and $y = (-x)^3 - (-x)^2$ on the same graph on your graphing calculator.

  From this example, the class will notice that the second graph in each case is the reflection of the first graph in the y-axis and that each point (x, y) on the original graph becomes the point (-x, y) on the second graph. Summarize that, in general, y = f(-x) is a reflection of y = f(x) in the y-axis.

- Consider the following points (0, -3) and (2, 1). Write an equation of the line passing through these points. Elicit $y = 2x – 3$. Reflect each point in the y-axis and write the equation of that line. Elicit $y = -2x – 3$. Challenge the class to find f(-x) if $f(x) = 2x – 3$. Elicit $y = -2x – 3$. (This may provide an alternate way to introduce/reinforce the concept of reflection in the y-axis.)

- Pose the following example to elicit the reflection of a graph in the y = x line:
  (e) Graph $y = 2x – 4$ and $y = x$ on the same graph.
  (f) Graph $y = \frac{1}{2}x + 2$ on the same graph and then explain how these two lines are geometrically related.

  Elicit that the second graph is a reflection of the first graph in the line y = x. Review that this is done by switching the y with the x in the first equation and solving for y. Summarize, in general, that x = f(y) is the reflection of y = f(x) in the y = x line. The concept of inverse need not be discussed at this point. It will be covered in a later lesson.

- The next part of this lesson deals with the tests for symmetry in an equation. Challenge the class to reflect $y = x^2$ in the y-axis. Elicit that the image is identical to the original equation. From this, challenge the class to identify what happens in a graph when y = f(x) is the same as y = f(-x). Summarize that if f(x) = f(-x) for a function y = f(x), then y = f(x) has an axis of symmetry on the y-axis. Furthermore, we say that y = f(x) is an **even** function. Challenge the class to determine if $y = x^4 – 3x^2 + 7$ is even. Ask the students to guess why this function is called even and elicit that for a polynomial, if all the exponents are even, it is called an even function. To introduce x-axis symmetry, sketch $x = y^2$. On the graph, elicit that in order for a relation to have x-axis symmetry, every time, (x, y) is on the graph, so is (x, -y). Summarize by stating the test for x-axis symmetry as: Substitute (x, -y) into the equation. If it simplifies to the original equation, it has x-axis symmetry.

- Challenge the class to graph $y = \frac{6}{x}$ and then reflect this graph in the origin. Review the concept of reflecting a graph in the origin and lead the class to realize that this graph is its own reflection in the origin. Select the point (2, 3) on $y = \frac{6}{x}$ and ask the class to identify the image after a point reflection in the origin. Elicit that (-2, -3) is the image of (2, 3) and that in general, (-x, -y) is the image of a point (x, y). Summarize that if both the x and y are negated in a given function, and the simplified expression is equivalent to the original function, then the graph has point symmetry about the origin. In this case, $(–y) = \frac{6}{(-x)}$ simplifies to $y = \frac{6}{x}$.
  Another way of testing is if f(x) = -f(-x), then y = f(x) has point symmetry about the origin. Furthermore, we say that y = f(x) is an **odd** function. Determine if $y = x^3 – 6x$ is an odd function. Elicit that it is because $y = -[(-x)^3 – 6(-x)] = x^3 – 6x$. Turn the graphing calculator upside down to see that $y = x^3 – 6x$ looks the same right side up as up side down. This is also a test for point symmetry about the origin and odd functions.

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