TOPIC 4.4: STRETCHING AND TRANSLATING GRAPHS

PERFORMANCE OBJECTIVES

The students will be able to:

- stretch and shrink graphs vertically and horizontally
- translate graphs, given changes in the equation of the graph

MATERIALS

Graphing calculator, overhead projector, prepared transparencies

STRATEGIES (This lesson may require two days.)

- Begin this lesson with the following exercise:
  Graph each of the following on the graphing calculator and describe how $y = x^2$, $y = (x - 2)^2$, $y = x^2 + 3$, are related. Elicit that $y = (x - 2)^2$ is a translation of $y = x^2$ two units to the right and that $y = x^2 + 3$ translates it three units up. Use the table below to summarize.

	$y = f(x) + k$ k units up	
$y = f(x + h)$ h units to the left	$y = f(x)$	$y = f(x - h)$ h units to the right
	$y = f(x) - k$ k units downs	

  Explain that this is the same concept that was used when graphing a circle in Math A, namely, $(x - 5)^2 + (y + 4)^2 = 9$ was a translation of $x^2 + y^2 = 9$ five units to the right and four units down. Have the students graph $y = | x |$, $y = | x + 5 |$, $y = | x | + 5$ using the above table.

- Begin the vertical stretch and shrink part of the lesson with the following task. If the class is working in groups, consider giving each group a different graph and have a presenter explain it to the class: Graph the functions $y = x^3 - 9x$, and $y = 2x^3 - 18x$. Describe how these two graphs are related algebraically and geometrically. Elicit that the second is a vertical stretch of the first by a factor of 2, i.e., the points above the x-axis are twice as high as the original points and those below the x-axis are twice as low as the original points. If the first equation can be thought of as $y = f(x)$, the second is $y = 2f(x)$. Compare the graphs of $y = \sin x$ and $y = 3 \sin x$. Again, elicit that the second is a vertical stretch of the first. If the leading factor is $\frac{1}{2}$, as in $y = \frac{1}{2} \sin x$, or $y = \frac{1}{2}x^3 - \frac{9}{2}x$, the resulting graph is a vertical shrink of the first.

  Use the table below to summarize the discussion:

If the graph of y = f(x) is changed to:	Then the graph of y = f(x) is

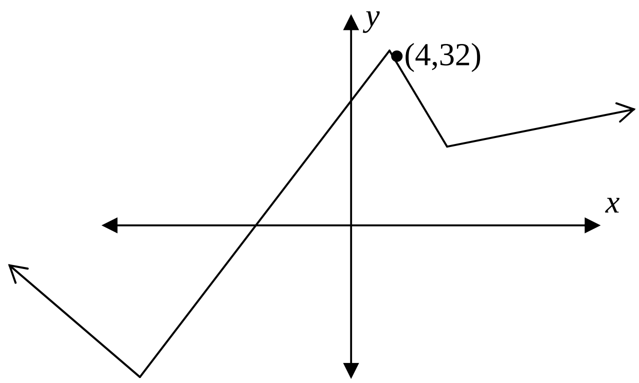
y = c f(x) and c >1	Stretched vertically by a factor of c.
y = c f(x) and 0 < c < 1	Shrunk vertically by a factor of c

- Begin the horizontal stretch and shrink part of the lesson with the following task. If working in groups, consider giving each group a different graph and have a presenter explain it to the class: Graph y = | x | and y = | 2x | and compare them. Then compare y = sin x and y = sin 2x and see how the graphs are affected by multiplying the variable x by 2. Elicit that both graphs are a horizontal shrink by a factor of 2. After graphing $y = \left|\frac{1}{3}x\right|$ elicit the horizontal stretch by a factor of 3. Use the table below to summarize:

If the graph of y = f(x) is changed to:	Then the graph of y = f(x) is
y = f(cx), and c > 1	Shrunk horizontally by a factor of c
y = f(cx), and 0 < c < 1	Stretched horizontally by a factor of c

- Place a prepared acetate on the overhead with the graph of y = f(x), as shown below. The local maximum of this graph is (4,32). Pose the following questions to the class: To what new point does this local maximum move if:
  (a) y = f (2x)
  (b) y = 2 f(x)
  (c) y = f (x – 2)
  (d) y = f (x + 2)?

  Elicit that for
  (a) (4, 32) moves to (2, 32).
  (b) it moves to (4, 64).
  (c) it moves to (6, 32)
  (d) it moves to (2, 32)



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