TOPIC 4.5: INVERSE FUNCTIONS

PERFORMANCE OBJECTIVES
Students will be able to:

- find the inverse of a function if it exists
- use a mapping diagram to determine if a composition of two functions is that of two inverses
- sketch the graph of the inverse
- determine if a function has an inverse by using the horizontal line test

MATERIALS
Graphing calculator

STRATEGIES

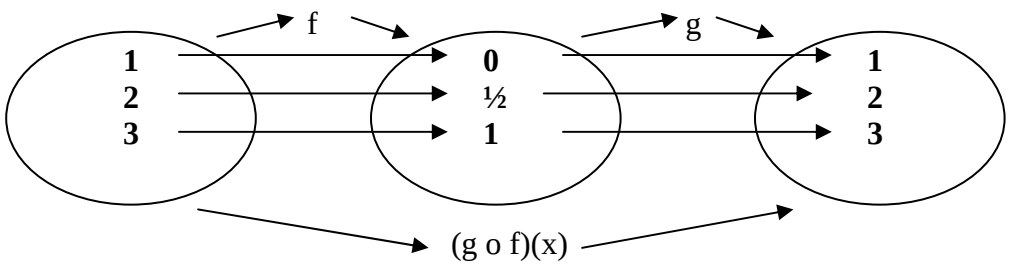
- Use the following Do Now to start the lesson:

  (a) Graph y = f(x) = $\frac{x-1}{2}$. (b) Graph g(x), a reflection of f(x) in the line y = x.

  (c) Write the equation of the g(x). (d) Explain how these two equations are related.

- Use this problem to review the rule for a line reflection in y = x, $(x, y) \xleftarrow{f(x)} (y, x)$, then (y,x) is the image after a line reflection in y = x. Elicit that the x and the y variables are interchanged in f(x) in the do now to get the equation for (c). After entering $\frac{x-1}{2}$ into the calculator, write the values of the table for x = 1, 2, and 3 on a diagram similar to what is shown on the bottom of this page. Let the students interchange the values for x and y in f(x), plot these points, and write the equation of the line that passes through them. Elicit that the equation g(x) = 2x + 1 is called the inverse of f(x) and is denoted by $f^{-1}(x)$. Examine the table at x = 0, ½ and 1. TBLSET (tableset), by pressing [2^{nd}] [WINDOW] on the TI-83, will need to be adjusted so ΔTbl = .5 Elicit that it is the reverse of the three points of f(x).

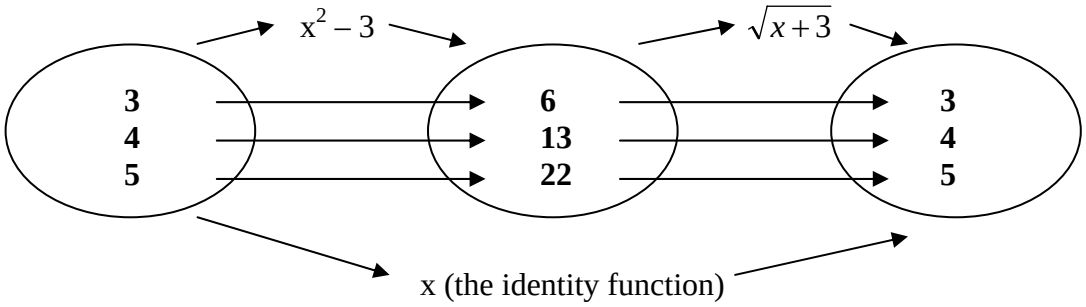
- Using the functions f(x) and g(x) in the do now, pose the following question: Evaluate f(g(3)) and g(f(3)). Discuss with the class the results of both answers are 3. Summarize by the following definition for the inverse of a function: Two functions *f* and *g* are called inverse functions if the following is true: (a) g(f(x)) = x for all *x* in the domain of *f* and (b) f(g(x)) = x for all *x* in the domain of *g*. The function that maps a variable into itself is called the Identity Function. In this case, the effect of composing a function with its inverse yields the same results as the identity function. Use the example of the do now to model how this works:



- Discuss with the class **the vertical line test** for checking if the relation is a function. On the graphing calculator, graph $y = x^2$. Instruct the students to reflect this graph in the line $y = x$. Discuss whether or not the reflection represents the graph of a function. Elicit that $x = y^2$ is not a function and why. Further elicit the criteria for an inverse to be a function: that is, any horizontal line intersects the graph of a function in at most one place. This implies that any vertical line will intersect the graph of the inverse only once. Define this method as **the horizontal line test**.

- In order to motivate why the term "inverse" is used, ask the class what is the inverse is for addition and elicit the following $(x) + (-x) = 0$. Similarly, ask the class for the inverse element for multiplication and elicit $(x) \bullet (\frac{1}{x}) = 1$ for all $x \neq 0$. Elicit that what is produced for both examples is the identity element for addition and multiplication respectively. Similarly, for the operation of composition, the composition of a function with its inverse function is the identity function.

- Summarize the lesson by asking the class to form the inverse function for $y = x^2 - 3$, $x \geq 0$. (Using $x \geq 0$ makes the inverse a function.) Elicit that the inverse is $y = \sqrt{x+3}$, $x \geq -3$. The domain restriction of $x \geq -3$ must be included in this example. Use the following mapping diagram to illustrate that the composition of these two functions is, in fact, the identity function.



The process of finding the inverse of a function is to interchange x and y in the original equation. Then solve for y. This new y-equation is the inverse of the original function and will produce a table of values that is the reverse of the original table.

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