TOPIC 4.6: FUNCTIONS OF TWO VARIABLES

PERFORMANCE OBJECTIVES

Students will be able to:

- understand the concept of a function with two variables
- graph functions of two variables on a two-dimensional graph

MATERIALS

Overhead projector, graphing calculator

STRATEGIES (This lesson is optional and may be deleted without loss of continuity.)

- Pose the following as a Do Now to get the lesson started:
  Given the formula for the perimeter of a rectangle, P = 2L + 2W
  (a) Name the dependent and independent variables.
  (b) Determine how many axes we would need to graph this function.
  (c) Discuss why this is different from other y = f(x) formulas we have discussed so far this year.

- Elicit that the answers to (b) and (c) are inter-related; we would need three axes to graph this function, whereas all the other functions discussed so far only needed two. This is because the perimeter function has three variables, L, W, and P, whereas, all the previous functions this year only had two variables. Elicit the need for a three-dimensional graph to plot the perimeter function. You may wish to have an overhead with a representation of a three-dimensional graph prepared. Explain that learning about three-dimensional graphs will occur later in the year; but inform the students that there is a way to graph a three-dimensional function on a two-dimensional graph.

- Elicit that holding one of the variables constant would make this three dimensional graph two-dimensional, and thus would allow it to be graphed on a two-dimensional graph. Ask the class to determine what would happen to the perimeter function if we were given that w = 1. Elicit P = 2L + 2; graph this on a (L, P) coordinate plot. Fix w = 2 and elicit P = 2L + 4, and then fix w = 3 and elicit P = 2L + 6. Graph these equations, labeling the corresponding fixed value of *w* to each line. Elicit that these lines are parallel, and that they are another way of representing a function of three variables.

- Ask the class to fix values of P and to graph the resulting equations on a (L, W) coordinate plot. Some examples of equations would be 2L + 2W = 12, 2L + 2W = 8, and 2L + 2W = 4.

- Elicit the following as a procedure to graph a function in two variables.

(1) Fix one of the variables by setting it equal to three different numerical constants.
(2) Graph the resulting two-variable functions on a single set of coordinate axes. Label the three fixed constants on the graph drawn.

- Pose the following example to practice the concepts that were just summarized to graph a three-dimensional function in two variables on a two-dimensional graph.
  The volume of a cylinder is given by the formula $V = \pi r^2 h$.

  (a) Sketch the graphs of V versus r for h = 1, h = 2, and h = 3.
  (b) Sketch the graphs of *h* versus *r* for $V = 36\pi$ and for $V = 72\pi$.
  (c) If *h* is held constant, what happens to *V* when *r* is doubled?

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