TOPIC 5.1: GROWTH AND DECAY: INTEGRAL EXPONENTS

PERFORMANCE OBJECTIVES

Students will be able to:

- define and apply integral exponents
- develop exponential functions for growth and decay
- determine whether a graph represents exponential growth or decay
- write the appropriate equation when given a growth or decay word problem

MATERIALS

Graphing calculator, blank and prepared transparencies, transparency markers, overhead projector

STRATEGIES

- Pose the following to the class, having them work either individually or in cooperative groups:
  (a) Simplify x + .10x.
  (b) How could you algebraically represent a 10% increase of x?
  (c) Suppose the costs of attending college increases 10% per year. Assuming the current year's cost is $15,000, how much will it cost to attend college ten years from now? How much did it cost to attend college ten years ago?

- From the above exercise students should recognize how to algebraically represent a growth rate of 10%. Make sure students realize this means each year the cost is 1.10 times the cost in the previous year. The second year's cost is 1.10x + .10(1.10x). Factoring the 1.10x we have $1.10x(1 + .10) = x(1.10)^2$. The third year's cost is $x(1.10)^2 + .10(x(1.10)^2) = x(1.10)^2(1.10) = x(1.10)^3$. Elicit from the students the equation for the cost as a function of time, C(t) = $15,000 $(1.10)^t$ and that at t = 0 (today) the cost is $15,000. Ask students to solve and graph the function and elicit from the class that the shape of the graph indicates exponential growth.

The chart below could be used to summarize the steps to help clarify the pattern.

Year	Cost of attending college	Cost of attending college in factored form
0	15000	15000
1	15000 + .10(15000)	15000(1.10)
2	15000(1.10) + .10(15000(1.10))	$15000(1.10)^2$
3	$15000(1.10)^2 + .10(15000(1.10)^2)$	$15000(1.10)^3$
10		$15000(1.10)^{10}$
t		$15000(1.10)^t$

- Ask the students to develop a function for the cost of a bike, assuming the cost has been decreasing 5% per year and the cost today is \$300. Ask students to graph the function. Discuss the difference between this graph and the graph representing college costs. Elicit that the formula would be adjusted $300(1 – .05)^t$. Introduce the term exponential decay. Formalize the model for growth and decay as $A(t) = A_0 (1+r)^t$, where A_0 is the initial amount, i.e. the amount at time t = 0, and r is the growth rate. If r > 0, then the initial amount grows exponentially. If -1 < r < 0, then the initial amount decays exponentially. (Note: If r < -1, the base of (1+r) is negative producing a model that is alternately positive and negative and therefore not an exponential model.) In order to cover the concept of decay, pose the following to the class: Suppose that a radioactive isotope decays so that the radioactivity present decreases by 15% per day. If 40 kg. are present now, find the amount present
  (a) 6 days from now, t = 6 and
  (b) 6 days ago, t = -6.

  Elicit that the function $A(t) = 40(1 – .15)^t$ models this decay and solves the problem, and that the solution to
  (a) is 15.1 kg. and
  (b) is 106.1 kg.

- Use the negative exponent in the previous example to have the class simplify by using all positive exponents: $\frac{x^5 + x^{-2}}{x^{-3}}$. Use examples, such as $(\frac{\mathbf{b}^2}{\mathbf{a}})^{-2}(\frac{\mathbf{a}^2}{\mathbf{b}})^{-3}$ and $(a^{-1} + b^{-2})^{-1}$ to review simplifying algebraic expressions using the laws of exponents, as well as, negative exponents, with the class.

Lesson plan by Carl Hudson and Tricia Weisberg