TOPIC 5.2: GROWTH AND DECAY: RATIONAL EXPONENTS

PERFORMANCE OBJECTIVES

Students will be able to:

- define and apply rational exponents
- write an expression in radical form that contains fractional exponents and vice versa
- apply the law of exponents to rational exponents
- solve exponential equations
- apply rational exponents to growth and decay problem
- solve rational equations

MATERIALS

Graphing calculator

STRATEGIES

- Pose the following problem to start the lesson. Solve and check the following exponential equation $27^{1-x} = (\frac{1}{9})^{2-x}$.

- Review the procedure for solving an exponential equation. Elicit that students need to find a common base for both sides of the equation and then set the exponents equal to each other. In this case, using the common base of 3, we have $(3^3)^{1-x} = (3^{-2})^{2-x}$. This simplifies to $3^{3-3x} = 3^{-4+2x}$ such that 3 – 3x = -4 + 2x which yields x = $\frac{7}{5}$. To check, we have $27^{-\frac{2}{5}} = (\frac{1}{9})^{\frac{3}{5}}$.

  Equating the bases we have $3^{3*(-\frac{2}{5})} = 3^{-2*(\frac{3}{5})}$ which yields $(3)^{-\frac{6}{5}} = (3)^{-\frac{6}{5}}$. Both of these expressions can be changed to a decimal by using a calculator.

- Use the check of the problem to review the meaning of rational exponents. Elicit that if an exponent is a fraction, its numerator tells us to what power to raise the base and the denominator tells us what root to take. In the expression $9^{\frac{3}{2}}$, for example, we have $9^3 = 729$, and then $\sqrt{729}$ = 27. Explain that in the order of operations, the root (denominator of the exponent) could be done first and the power (the numerator of the exponent) second. In evaluating $9^{\frac{3}{2}}$, we also could have done $\sqrt{9}$ = 3 and $3^3 = 27$. The negative symbol in an exponent means that we should take the reciprocal of the answer. So if we have, $9^{-\frac{3}{2}}$ we simplify as before and at the end, take the reciprocal of the answer 27 to get $\frac{1}{27}$ or $.\overline{037}$.

To show that the laws of exponents still apply even if they are rational, challenge the class to simplify $(8a^{-6})^{-\frac{2}{3}}$. Elicit that this may be simplified in several ways. including: $(8)^{-\frac{2}{3}}(a^{-6})^{-\frac{2}{3}} = (\sqrt[3]{8})^{-2}(a^4) = (2)^{-2}a^4 = \frac{1}{4}a^4$. Select an example like, $a^{\frac{1}{2}}(a^{\frac{3}{2}} - 2a^{-\frac{1}{2}})$, and elicit that all the rules of algebra apply if the exponents are rational.

- For a medial summary of working with rational exponents, have students solve the following problem using the growth and decay formula: In the city, the value of a house increases at a rate of 16% annually. The present value of a single family house is $150,000. Find the value of the house in two years and six months. Elicit that by using the basic growth formula, we have $A(t) = A_o(1 + r)^t = 150000(1 + .16)^{2.5}$. The exponent in this expression is a rational number and may be evaluated to yield an answer of $217,388.

In order to review the solution of a rational equation, pose the following: A new luxury car model could be purchased in 1997 for $85,000. In 2001 the same model costs $100,000. What is the annual growth rate of the cost of a luxury car (to the nearest hundredth of a percent)? Use the formula $A(t) = A_o(1 + r)^t$ to get $100{,}000 = 85{,}000(1 + r)^4$. Dividing both sides by 85,000, we have $1.18 = (1 + r)^4$. Taking the fourth root of both sides we have: $(1.18)^{\frac{1}{4}} = 1 + r$ and $1.0422 = 1 + r$, and therefore $r = 0.0422$. If you purchased the car in 1997, the growth rate in the manufacturer's cost was 4.22%. Finally, pose: If the same $85,000 car was purchased in 1997 and depreciates 30% per year, how much will it be worth in 4 years and 6 months after the date of purchase? Elicit that by using the basic formula again, we have $A(t) = A_o(1 - r)^t = 85{,}000(1 - 0.30)^{4.5} = 85{,}000(0.70)^{4.5} = 17{,}074.98$, If the car depreciates by 30%, it will be worth $17,0745 in 4 years and 6 months

- Next, elicit how to simplify the expression into a single fraction: $(2x+1)^{\frac{2}{3}} - 4(2x+1)^{-\frac{1}{3}}$. This may be done by writing the expression with all positive exponents, i.e., $(2x+1)^{\frac{2}{3}} - \dfrac{4}{(2x+1)^{\frac{1}{3}}}$.

A common denominator of $(2x+1)^{\frac{1}{3}}$ may be found yielding $\dfrac{(2x+1)^{\frac{2}{3}} * (2x+1)^{\frac{1}{3}}}{(2x+1)^{\frac{1}{3}}} - \dfrac{4}{(2x+1)^{\frac{1}{3}}}$.

The numerator of the first fraction simplifies to $(2x+1)^{\frac{3}{3}}$ which is (2x + 1). Then, the difference of these fractions is $\dfrac{2x-3}{(2x+1)^{\frac{1}{3}}}$. This example could have also been simplified by factoring out $(2x+1)^{-\frac{1}{3}}$ from the original expression. We then have $(2x+1)^{-\frac{1}{3}}[(2x + 1) - 4]$, which simplifies to $(2x+1)^{-\frac{1}{3}}(2x - 3)$ which is the same as the previous solution.

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