TOPIC 5.3: EXPONENTIAL FUNCTIONS

PERFORMANCE OBJECTIVES

Students will be able to:

- define and use exponential functions
- use exponential functions to describe growth and decay
- use exponential functions to solve real life applications
- identify patterns in the graphs of exponential functions

MATERIALS

Graphing calculator, overhead projector

STRATEGIES

- Ask the students to complete the following Do Now:
  Suppose that t hours from now the population of a bacteria colony is given by
  $P(t) = 90(100)^{\frac{t}{8}}$
  (a) What is the population when t = 0?
  (b) How long does it take for the population to be multiplied by 100?

  Have the students graph the function and elicit that when t = 0 the population of the bacteria colony is 90 (meaning it is at its initial state.) Elicit that the graph of the function is increasing because as *t* gets bigger $(100)^{\frac{t}{8}}$ also gets bigger. The following table shows the class how the change in *t* affects the theoretical population of the colony:

T	0	1	2	3	4	5	6	7	8
$90(100)^{\frac{t}{8}}$	90	159.3	2884.58	506.07	900	1600.47	2846.07	5061.06	9000

  Elicit that the 8 in the denominator of the exponent means that it takes the colony 8 hours to multiply by 100 (100 times bigger than the initial state).

- Pose the next problem to the class:
  A colony of bacteria decays so that the population t days from now is given by
  $A(t) = 1000\,(\frac{1}{2})^{\frac{t}{4}}$.
  (a) What is the amount of bacteria present when t = 0?
  (b) How much bacteria will be present in 4 days (t = 4)?
  (c) What is the half-life of the bacteria colony?
  (d) Graph the function when t > 0
  (e) How long will it take before the colony is dead?
- Have the students graph the function when t = 0 the population of the bacteria colony is 1000. When t = 4 (day 4) the population of the colony is decreased by ½ therefore the colony

is half its initial state. The graph of the function is decreasing because as t increases the value of $(\frac{1}{2})^{\frac{t}{4}}$ decreases. Prepare a similar table as before on acetate for this problem:

t	0	1	2	3	4
$(1000)(\frac{1}{2})^{\frac{t}{4}}$	1000	841	707	595	500

Elicit how the change in *t* affects this graph differently from the previous example and makes the exponential function a decreasing one. Also elicit that the denominator of the exponent is 4 and indicates that the population is halved in 4 days.

- Review the basic exponential functions by having the students graph the following on their calculator: $f(x) = 3^x$, $g(x) = 5^x$ and $h(x) = 10^x$. Elicit that for functions of the type $f(x) = b^x$ for $b > 1$, the graphs rise to the right and as *b* gets larger the graphs get steeper. In addition, all the graphs have the point (0, 1) as their y-intercept.

- Now have the students graph on the calculator $p(x)= (\frac{1}{2})^x$, and $r(x) = (\frac{2}{3})^x$. Elicit that if the base of the exponential function is a proper fraction, the graph decreases from left to right.

- The following set of exercises reviews the effects of certain transformations on exponential graphs. Graph $f(x) = 2^x$ and $g(x) = (1/2)^x$. Elicit from the class that these two graphs are reflections of each other in the y-axis. Also graph $y = 2^{-x}$, using the [–0] tool on the [Y=] menu, and elicit that $y = (1/2)^x$ is the same as $y = 2^{-x}$. Review that $y = f(-x)$ is the reflection of $y = f(x)$.

- Graph $h(x) = 4^x$ and $k(x) = 4^{x/2}$. Elicit from the class that $k(x) = 4^{x/2}$ is the graph of $h(x) = 4^x$ stretched horizontally by a factor of 2.

- Graph $p(x) = 2^x$ and $q(x) = 2^{x-1}$. Elicit from the class that the graph $q(x) = 2^{x-1}$ is the graph of $p(x) = 2^x$ shifted one unit to the right

- To summarize the lesson have the class do the following exercise:
  The half-life of a radioactive isotope is 4 days. If 3.2 kg are present now, how much will be present after: (a) 4 days?, (b) 8 days?, (c) t days? To find the amount of the element after 4 days: Write the equation needed to model and solve the problem, i.e., $f(t) = 3.2(\frac{1}{2})^{\frac{t}{4}}$ and substitute t = 4 to get that on day 4 there are 1.6 kg. To find the amount present on day 8: find f(8) = .8 kg. To find the amount present on *t* days, find $f(t) = 3.2\ (1/2)^{t/4}$.

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