

TOPIC 5.5: LOGARITHMIC FUNCTIONS

PERFORMANCE OBJECTIVES

Students will be able to:

- to define common and natural logarithms
- to convert from logarithmic to exponential form
- to define logarithms as inverses of exponential functions
- to solve equations using the rules for logarithms

STRATEGIES

- Pose the following problem to get the lesson started:
 - (a) Create a table for $y=2^x$ using the domain -3 to 3.
 - (b) Create a table of ordered pairs that are images of those pairs in the above table under a reflection in the line $y = x$.
 - (c) Sketch the graph of $y = 2^x$ and the graph of its image after this reflection in the line $y = x$.
 - (d) What is the equation of the new graph (the image)?
- Use this problem to elicit the concept of inverse functions and to guide students to $x = 2^y$ and then to $y = \log_2 x$. Review the concept that the word logarithm means exponent, and that what we are really doing is looking for the exponent for the base 2 needed to obtain x . Have students define the logarithmic equation and function in their own words. Using the graphs, have students discuss the domain and range of each of the functions. Draw the graph of $y = \log_2 x$. Elicit from students that the range of the exponential function becomes the domain of the logarithmic function. Ask the class to find the $\log(-5)$ and then ask why the calculator gives an error message. In this discussion, elicit that the composition of these two inverse functions, in either order, is the identity function. $(f \circ g)(x) = \log_2(2^x) = x$ and $(g \circ f)(x) = 2^{\log_2 x} = x$. This may be done on a prepared overhead transparency.
- Pose the following to the class to contrast the difference between common logarithms and natural logarithms in order to get the students working with these two different kinds of logarithms.
- Elicit that a logarithm having e as a base is called a natural logarithm and written as $\ln x$. Have students find the value of x to the nearest hundredth:
 - (a) $10^x = 75$
 - (b) $e^x = 75$.Instruct the class to rewrite each into their logarithmic form:
 - (a) $\log_{10} 75 = x$
 - (b) $\ln_e 75 = x$.

Use a calculator to find $\log 75 = 1.875$ and $\ln 75 = 4.317$ and discuss why are they are different. Elicit and review the graphs of $y = \log x$ and $y = \ln x$.

- Pose the following practice problems to the class and review the concepts of converting from logarithmic form to exponential form, i.e., $a^y = x$ iff $y = \log_a x$,

(a) Change from logarithmic form to exponential form and vice versa:

- (1) $\log_6 x = 2$
- (2) $\log 1000 = 3$

(b) Find each logarithm without using a calculator:

- (1) $\log 100$
- (2) $\log_3 243$
- (3) $\log_4 \frac{1}{64}$
- (4) $\ln e$
- (5) $\ln e^2$
- (6) $\ln \frac{1}{e}$

(c) Use a calculator to approximate each logarithm:

- (1) $\log 6800$
- (2) $\ln 2.8$

- In order to apply the concepts of converting from logarithmic form to exponential form, pose the following to the class: Solve each equations for x:

- (a) $\log_x 121 = 2$
- (b) $\log x = -1.2$
- (c) $\log_6 (\log_2 x) = 1$
- (d) $\ln (x - 2) = 1$

In solving each of these examples, review the basic conversion from logarithmic form to exponential form.

- (a) $x^2 = 121$.
- (b) $10^{-1.2} = x$.
- (c) we need to apply the concept twice, first to get $6^1 = \log_2 x$ and again to get $2^6 = x$.
- (d) $e^1 = x - 2$.

- Review the impact of transformations on graphs in the following example:
Sketch each and give the domain, range, and zeros of the functions $y = (\log x) + 3$, $y = \log (x + 3)$ and $y = |\log x|$.

- Discuss common logarithms and their use in applications involving the perception of sound. Every sound has an intensity level due to the power of the sound wave. Several familiar sounds are given intensity levels in units called decibels (dB). The formula that relates a sound's decibel level I of a noise is as follows: decibel level of $I = 10 \log \frac{I}{I_0}$. The sound of a loud stereo has intensity 10^8 times the intensity of a barely audible noise. Its decibel level can be found by substituting $I = 10^8 I_0$ into the above formula. So, for a loud stereo, $I = 10 \log \frac{10^8 I_0}{I_0} = 10 \log 10^8 = 10 \cdot 8 = 80$. The decibel levels of some other sounds include a whisper which has a level of 10 dB, soft recorded music which has a level of 40 dB, two people in conversation which has a level of 60 dB, a subway train which has a level of 100 dB, and a jet at take off of 120 dB. By using this formula, we can show that when two loud stereos are playing loudly, that the sound intensity is found by using the formula $10 \log \frac{2 \times 10^8 I_0}{I_0} \approx 83$ dB. Since the decibel level of one stereo is 80 dB, there is only about a 3 dB increase in the decibel level when two are playing. A decibel scale is an example of a logarithmic scale. Other examples of logarithmic scales are scales that measure acidity and brightness.

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