TOPIC 5.6: LAWS OF LOGARITHMS

PERFORMANCE OBJECTIVES

Students will be able to:

- prove the laws of logarithms
- apply the laws of logarithms
- use the laws of logarithms to solve equations
- change expressions from exponential form to logarithmic form and vice-versa
- solve logarithmic equations
- sketch the graph of a logarithm function

STRATEGIES

- Start the lesson with the following Do Now:

  Convert each into powers of 10 and simplify: $\frac{(24)(\sqrt{2})}{(.5)^4}$

  From this example elicit from the students that in order to find the power to which 10 has to be raised in order to get 24 you have to find log 24. So $10^{1.3802} = 24$, because log (24) = 1.3802. Using the same procedure express 2 as $10^{.3010}$, and .5 as $10^{-.3010}$.

  Substituting each of these into the original problem, we now have: $\frac{(10)^{1.3802}\sqrt{(10)^{.3010}}}{(10^{-.3010})^4}$.

  Elicit that in order to find the square root of $\sqrt{(10)^{.3010}}$, we will need to divide the exponent .3010 by 2 to get .1505. The problem now becomes: $\frac{(10)^{1.3802}(10)^{.1505}}{(10^{-.3010})^4}$.

  Elicit that since the base of both factors of the numerator are both 10, that we need to add the exponents of 1.3802 and .1505. The problem now simplifies to the following: $\frac{(10)^{1.5307}}{(10^{-.3010})^4}$. In order to simplify the denominator of this fraction we use the law for raising a power to a power and multiply the exponents to get $\frac{(10)^{1.5307}}{10^{-1.204}}$. Finally in order to divide, we subtract the exponents to get: $(10)^{2.7347}$ = 542.875197. If the original problem, $\frac{(24)(\sqrt{2})}{(.5)^4}$, had been calculated on a calculator, the same answer would have been obtained.

- From all of the logarithm applications in this example, elicit and summarize the law of logarithms on the board:

  1. $\log_b M \bullet N = \log_b M + \log_b N$
  2. $\log_b \frac{M}{N} = \log_b M - \log_b N$
  3. $\log_b M^k = k \log_b M$, for any real number k

  The proof of the first theorem may be done in class as follows:
  Let $\log_b M = x$ and $\log_b N = y$. Then $M = b^x$ and $N = b^y$. $M \bullet N = b^x \bullet b^y = b^{x+y}$.
  Therefore, $\log_b M \bullet N = x + y$ and $\log_b M \bullet N = \log_b M + \log_b N$.

- Pose the following applications to the class. Challenge the class to apply the laws of logarithms to these three examples.

  (a) Express $\log_3 15 + \log_3 4 - \log_3 2$ as a single logarithm using the laws of logarithms.
  (b) Solve the given equation $\log_2 (x + 2) + \log_2 5 = 4$.
  (c) If $\log_8 3 = r$ and $\log_8 5 = s$, express the $\log_8 75$ in terms of *r* and *s*

  Review the following solutions with the class.

(a) $\log_3 30$	Let $\log_3 30=x$ and rewrite into exponential form. [the numerical value of $\frac{\log 30}{\log 3}$ is 3.096]
(b) $\log_2 5(x + 2) = 4$	$2^4 = 5x + 10$, $x = \frac{6}{5}$.
(c) $\log_8 75$	$= \log 5^2 \bullet 3 = 2 \log_8 5 + \log_8 3 = 2s + r$.

- Review with the students the laws for transformations using the following example.

  (a) Predict how the graph of $y = \log_b (1/x)$ is related to the graph of $y = \log_b x$.
  (b) Check your prediction by graphing $y = \log_b x$ and $y = \log_b (\frac{1}{x})$ using the graphing calculator.

Lesson plan by Inna Buff, Giselle Nasif and Judy Mui