TOPIC 6.1 COORDINATE PROOFS

PERFORMANCE OBJECTIVES:

Students will be able to:

- prove two nonvertical lines are parallel using their coordinates
- prove two lines are perpendicular using coordinate geometry
- prove two line segments are congruent using coordinate geometry
- prove two lines bisect each other
- do general coordinate geometry proofs

MATERIALS

Graph paper

STRATEGIES

- This lesson covers coordinate geometry proofs and allows students to do proofs using the distance, midpoint and slope formulas. Pose the following to get the lesson started:
  Sketch the following on graph paper: A (0, 0) B (7, 0) C (10, 5) D (3, 5).
  Prove that ABCD is a parallelogram.

- Elicit from the class that in order to prove that ABCD is a parallelogram, one pair of opposite sides needs to be congruent and parallel. Elicit from the class how to prove congruency of two line segments by the distance formula. Elicit from the class that to prove the sides are parallel they have to have the same slope. Then ask the class to draw the diagonals of ABCD. Instruct the class to prove that the diagonals of a parallelogram bisect each other by the midpoint formula. With the class, review the distance formula: d = $\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$, the slope formula: m = $\frac{y_2 - y_1}{x_2 - x_1}$, and the midpoint formula: $(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2})$. Review how to determine when two lines are parallel and perpendicular by using slopes.

- Summarize during this lesson the methods commonly used in coordinate proofs:
  (1) To prove line segments equal, use the distance formula to show that they have the same length.
  (2) To prove nonvertical lines parallel, show that they have the same slope.
  (3) To prove lines perpendicular, show that the product of their slopes is –1.
  (4) To prove that two line segments bisect each other, use the midpoint formula to show that each segment has the same midpoint.
  (5) To show that lines are concurrent, show that their equations have a common solution.

- In order to motivate the nature of a coordinate proof, pose the following question: Prove that the midpoint of the hypotenuse of a right triangle is equidistant from the three vertices. As in proofs in geometry, we need to identify the given information and the item we wish to prove. Draw, on a set of coordinate axes, a right triangle whose vertices are located at the following

points: A (a, 0), B (0, b) and C (0, 0). Elicit that we know this is a right triangle by the structure of the coordinate axes. We are given that ∠C is a right angle and that a point M is the midpoint of AB. We need to prove that MC = MA. (We already know that MB = MA). Find the coordinates of M using the midpoint formula to get $(\frac{a}{2}, \frac{b}{2})$. Use the distance formula on MC to get $\sqrt{\frac{a^2}{4} + \frac{b^2}{4}}$. Use the distance formula on MA to get the same result. Finish the proof by writing "Therefore, MC = MA, and since MA = MB, we now have that MA = MB = MC."

- Pose a second example to assess understanding: Prove that the median of a trapezoid is parallel to the bases and has length equal to the average of the lengths of the bases. Plot the trapezoid in a convenient location O (0, 0), P (a, 0), Q (d, c) and R (b, c). Elicit that the ordinates of Q and R must be the same and that the ordinates of O and P must also be the same. We are "Given: OPQR is a trapezoid. Points M and N are midpoints of OR and PQ respectively." We need to "Prove: (1) MN || OP and (2) MN = $\frac{OP + RQ}{2}$." Find the midpoints M $(\frac{b}{2}, \frac{c}{2})$ and N $(\frac{d+a}{2}, \frac{c}{2})$. The slope of RQ, MN and OP are all zero and therefore parallel. Use the distance formula to show MN = $\frac{d+a}{2} - \frac{b}{2}$ and that $\frac{1}{2}$(OP+RQ) is equal to the same expression and therefore, MN = $\frac{1}{2}$(OP+RQ).

- Summarize the lesson by listing the steps needed to do a coordinate geometry proof.
  (1) Draw a diagram in a convenient location on a coordinate axes. Label the coordinates of the vertices.
  (2) List the GIVEN information and identify what is to be PROVEN.
  (3) Use the coordinate geometry formulas of slope, midpoint and distance where needed.
  (4) Explain what your formulas show in separate sentences.

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