TOPIC 6.2: EQUATIONS OF CIRCLES

PERFORMANCE OBJECTIVES

Students will be able to:

- identify the radius and coordinates of the center of a circle when given the equation of a circle
- use the features on the graphing calculator to graph equations of circles
- find the points of intersection of a line and a circle algebraically
- apply the equation of a circle to solve maximum/minimum problems

MATERIALS

Graphing calculator

STRATEGIES

- Start the lesson by reviewing the standard forms for the equation of a circle. Pose the following Do Now problems:
  (a) What is the center and radius of the circle with the equation $(x - 3)^2 + (y + 7)^2 = 19$?
  (b) What is the equation of a circle if the center is at the origin?
  (c) Find the center and radius of a circle with an equation $x^2 + y^2 - 6x + 14y + 58 = 0$.
  (d) Graph the equation from part (c) on a set of coordinate axes.

- The discussion of the do now should include that the equation of a circle whose center is (h, k) and radius r is $(x - h)^2 + (y - k)^2 = r^2$ and a circle whose center is at the origin, (0, 0), and radius r is $x^2 + y^2 = r^2$. Elicit from students that when the equation of the circle is written in a form such as part (c), completing the squares in x and y will result in the equation being rewritten in the center-radius form.

- Incorporate the graphing calculator into the lesson asking students to graph the equation $x^2 + y^2 = 4$ on the graphing calculator. Since the graphing calculator can only graph equations in two variables that are solved for y, the equation must be rewritten. Elicit from the students that there will always be two equations in terms of y for each circle, i.e., $y = \pm\sqrt{9 - x^2}$, the positive representing the top half of the circle, the negative the bottom portion. Show the students that once you enter the equation for the top portion of the circle as Y_1, you do not need to retype the entire equation for the bottom portion of the circle. Let $Y_2 = -Y_1$ using the VARS, and Y-VARS features to do this.

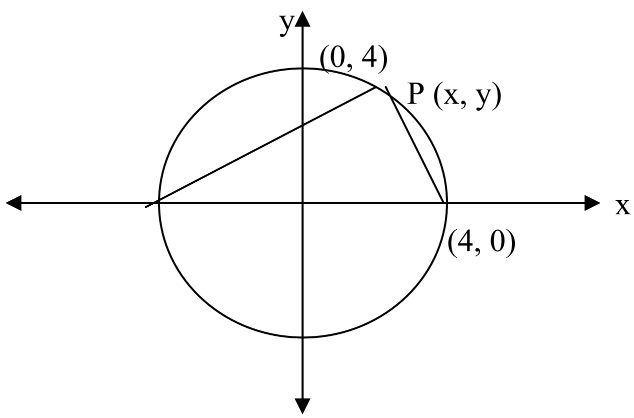
- When students graph the equation, it will most likely appear to be an ellipse. Explain that the graph looks like an ellipse because the pixels on the TI-83 do not represent equal width and height. To adjust the graph, press [ZOOM], [5]. The new graph should appear as a circle. [ZOOM], [5] is the feature called "zoom square" [ZSquare] and makes the metric spacing of the x-axis and y-axis the same.

- Continue by posing the following problem: What are the coordinates of the points where the line $y = 2x - 2$ and the circle $x^2 + y^2 = 25$ intersect? Allow students to work together for a few minutes. Then go over each step of the algebraic solution:
  (1) Solve the linear equation for y in terms of x (if necessary).
  (2) Substitute the linear expression for y in the circle's equation and solve the resulting quadratic equation.
  (3) Substitute each real x solution in the linear equation to get the corresponding value of y.
  (4) Each point is an intersection point.
  Elicit from students that there can never be more than two points of intersection. If there are no points of intersection, solving the quadratic equation (from step 2) will result in imaginary roots.

- Confirm the answer to the problem above by checking the solution against the graphing calculator. Key in the linear equation into the TI-83 as Y_3. If students are not familiar with the CALC, [intersect] feature, go over it with the class.

- Additional problems may be given if practice is needed in finding the intersection of an equation of a circle and a linear equation. Use the following equations for practice:
  (a) $x + y = 23$, $x^2 + y^2 = 289$ (2 points of intersection)
  (b) $x + 2y = 10$, $x^2 + y^2 = 20$ (1 point of intersection, line is tangent to the circle)
  (c) $2x - y = 7$, $x^2 + y^2 = 7$ (no points of intersection)

- Summarize the lesson by having students incorporate their knowledge of the equation of a circle into the following maximum/minimum problem:
  A triangle is inscribed in a semicircle as shown below.



(a) Find the equation for the semicircle. Elicit $y = \sqrt{16 - x^2}$

(b) Write a function A(x) for the area of the triangle. $A(x) = \frac{1}{2}(8)\sqrt{16 - x^2}$

(c) What is the domain of the function from part (b)? $-4 \leq x \leq 4$

(d) Graph $y = A(x)$ on the graphing calculator.

(e) Use the graph from part (d) to find the value of x that maximizes A(x). $x = 0$

(f) What is the maximum value of A(x)? $A = 16$

Interpret this to mean that when $x = 0$, the point on the semicircle is (0, 4) producing a triangle whose area is 16 square units.

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