TOPIC 6.6: SYSTEMS OF SECOND DEGREE EQUATIONS

PERFORMANCE OBJECTIVES

Students will be able to:

- solve systems of 2^{nd} degree equations algebraically
- solve systems of 2^{nd} degree equations by a graphing calculator
- graph an equilateral hyperbola in the form of xy = k or (x – h)(y – k) = k

MATERIALS

Graphing calculator

STRATEGIES (This lesson may require two days.)

- Consider starting the lesson with the following Do Now:
  Graph the following system of equations using your graphing calculator:
  (a) xy = 6
  (b) $x^2 + 4y^2 = 64$.

  Elicit the shape of each equation. The ellipse $x^2 + 4y^2 = 64$ can be transformed to its standard form $\frac{x^2}{64} + \frac{y^2}{16} = 1$. A rough sketch will indicate that there are four solutions to this system of equations because there are four points of intersection. The rough sketch will not necessarily give a good estimation of the actual points of intersection. Elicit that the hyperbola and ellipse may be entered into the graphing calculator as [Y_1] = $\frac{6}{x}$ (hyperbola), and [Y_2] = $\sqrt{16 - \frac{x^2}{4}}$ (top half of the ellipse), [Y_3] = $-\sqrt{16 - \frac{x^2}{4}}$ (bottom half of the ellipse). To find the point of intersection in the first quadrant, on the TI-83 graphing calculator, press the following set of keystrokes: [2^{nd}], [TRACE], [5], [ENTER]. The calculator automatically selects the first curve Y_1, the hyperbola. Move the cursor near the point of intersection in quadrant I and press [ENTER]. The calculator automatically selects Y_2, the top half of the ellipse when [ENTER] is pressed. The cursor is still near the point of intersection. Press [ENTER] again and the calculator calls for a guess. Press [ENTER] a third time and the calculator gives the coordinates of the points of intersection, (1.5, 3.9). Repeat this on any of the other points of intersection and those coordinates will be given: (-1.5, -3.9), (7.9, .76), and (-7.9, -.76). The point symmetry about the origin of this graph is reinforced by these four answers.

- The algebraic solution of the above system of equations should also be done at this point. Solving the hyperbola for y and substituting it into the ellipse gives a **bi-quadratic** quartic equation $y^4 - 16y^2 + 9 = 0$. This can not be factored, so the quadratic formula needs to be used. The variable is y^2 for this bi-quadratic equation with a = 1, b = -16 and c = 9. We have: $y^2 = \frac{16 \pm \sqrt{16^2 - 4 \bullet 9}}{2} = 8 \pm \sqrt{55} = 15.4 or .58$. So y = ± 3.9 or ± .76. Substituting these values for y back into xy = 6, we have the four algebraic solutions to the system that match the four points of intersection of the graphic solution above.

- The discussion of the do now should also review the graph of equilateral hyperbolas. Have the class graph the following equations in their calculators as well xy = 12 and xy = -2. Elicit that the asymptotes of hyperbolas are the x and y-axes.

  Challenge the class to graph (x – 2)(y – 5) = 6 and elicit that this is merely a translation of xy = 6, 2 units to the right and 5 units up. The center of this hyperbola is (2, 5) and the asymptotes are x = 2 and y = 5. Summarize the discussion with "If xy = k and k > 0, the graph is located in quadrants I and III. If k < 0, the graph is located in quadrants II and IV.

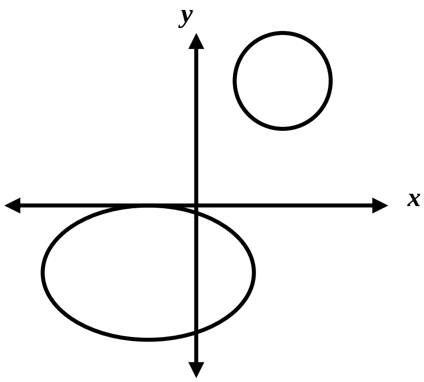
- Have the class solve the following system $2x^2 + y^2 = 4$ and $x^2 - 2y^2 = 12$ algebraically. Solving for x we have x = ± 2. Substituting this into either equation yields y = ± 2*i*. Theoretically, we have the four points of intersection (2, 2i), (2, -2i), (-2, 2i), and (-2, -2i). Have students graph this system in their calculators to interpret what these findings mean. What they discover is that the equations produce an ellipse and a hyperbola which do not intersect. Summarize the discussion: "If the algebraic solution to a system of equations produces an imaginary point of intersection, the graphs do not intersect."

- Elicit that the first system of equations had 4 solutions while the second no solutions. Challenge the class to identify how many solutions are possible when solving a pair of quadratic equations. Using circles, ellipses, hyperbolas, and parabolas, have the class draw pairs of these equations with no solutions, one solution, two solutions, three solutions, and four solutions. Put these drawings on an acetate sheet and share with the class. Examples of each possibility can be found on the next page. These can be made into a transparency to show the class if time is running short.

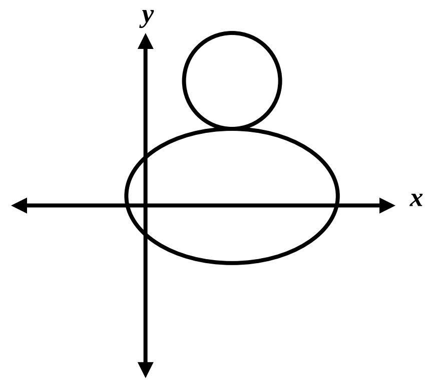
- To summarize with the class, review the methods used to solve systems of 2^{nd} degree equations and when to use each.
  (1) Solve graphically by graphing calculator, when the equations are solved for y or can easily be solved for y.
  (2) Solve algebraically by substitution, when one equation is solved for either x or y, and it is easy to substitute into the other equation.
  (3) Solve algebraically by addition - subtraction when the 2 equations are either a origin centered circle, ellipse or hyperbola.

Lesson plan by Andrea Hoagland, Dvora Geller and Nancy Fleming

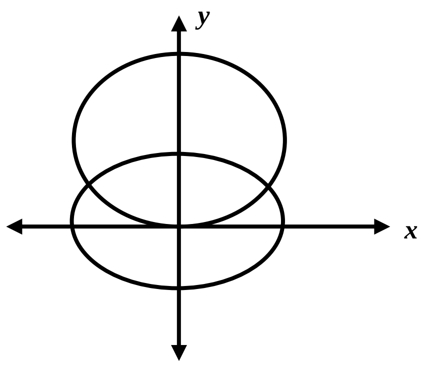
Different Kinds of Quadratic – Quadratic Systems



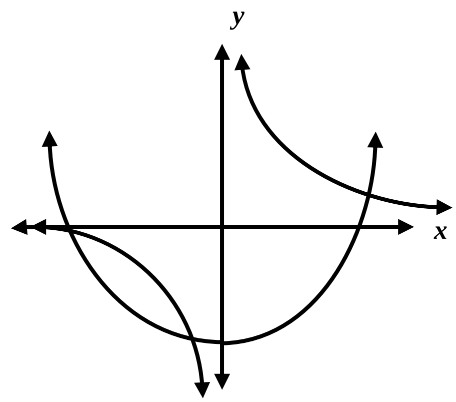
No solutions



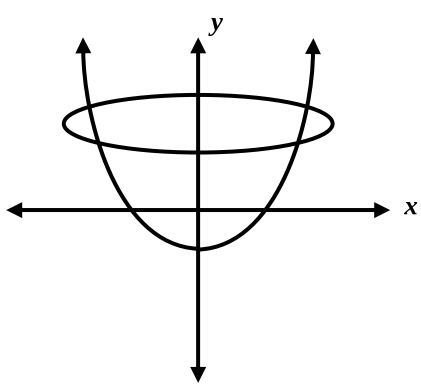
One solution



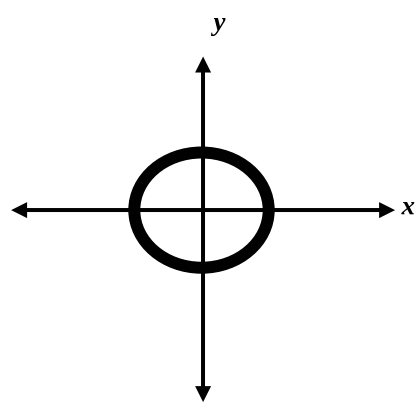
Two solutions



Three solutions



Four solutions



Infinitely many solutions