TOPIC 7.1: MEASUREMENT OF ANGLES

PERFORMANCE OBJECTIVES

Students will be able to:

- find the measure of an angle in either degrees or radians
- convert between degrees and radians
- determine whether two angles are co-terminal
- draw a positive or negative angle in standard position
- identify the initial and terminal side of an angle

MATERIALS

TI-83 calculator, protractor, overhead projector

STRATEGIES

- Pose the following to the class:
  (a) Convert 180° to radians by pressing the following keystrokes on your TI-83 calculator (make sure your calculator is in radian mode by pressing mode and highlighting radians): [1], [8], [0], [2nd], [MATRIX], [1], [ENTER] or (on TI-83PLUS): [1], [8], [0], [APPS], [1], [ENTER]
  (b) What does this number represent?
  (c) Write a formula to convert degrees into radians.

- Define radian measure by putting the following on a transparency:
  When an arc of a circle has the same length as the radius of the circle, the measure of the central angle is by definition 1 radian. Compare the diagram of a radian to that of an equilateral triangle and challenge the class to estimate that 1 radian is slightly less that 60°, and let the class know that it is approximately 57.3°. Elicit that 2 radians are approximately 114.6° and that 3 radians are approximately 171.9°. From this discussion challenge them to name how many radians are in 180°? From this elicit the relationship that π radians = 180°. From this generalize that the radian measure for 30°, 45°, 60°, and 90° can mentally converted. Explain to the students that these angle measurements appear often in trigonometry and it will be helpful for the students to be able to recognize these equivalents without using a calculator.

- Give students a brief history about the Babylonian numeration system. The convention of having 360 degrees in one revolution can be traced back to the fact that the Babylonians used a system that was based on the number 60. Theory suggests that the Babylonians divided an angle of an equilateral triangle into 60 equal parts, which became known as degrees. Six equilateral triangles arranged into a regular can be placed inscribed in a circle, 1 revolution contains 6 x 60 = 360 degrees.

- In order to motivate the formula S = θr, pose the following to the class: Find the length of an arc of a circle whose central angle is 2 radians and whose radius is 5. Elicit that answer is 10 using the definition of a radian.

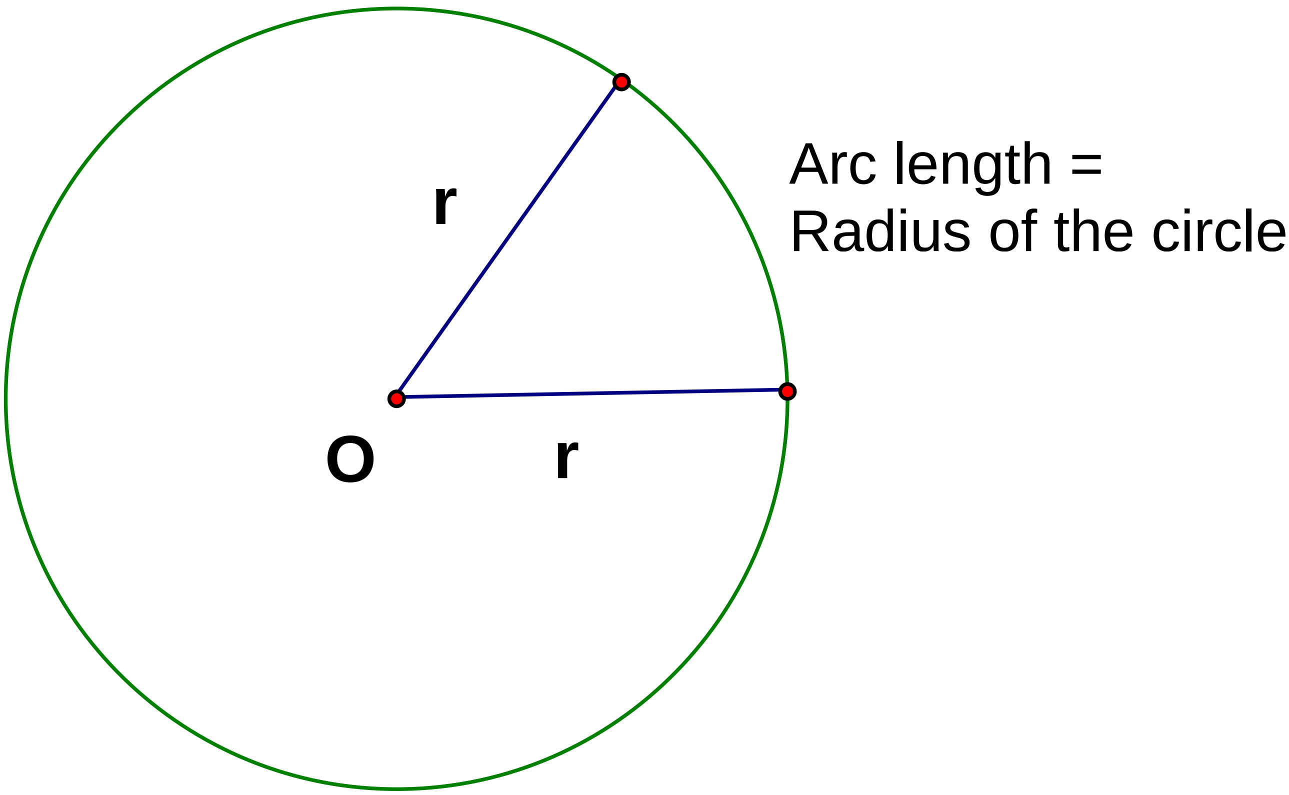
- In order to review many of the basic topics from the previous course, pose the following to the class: ask the class to draw a 30° angle in the first quadrant of a coordinate plane with the initial ray on the x-axis. Review the terms initial and terminal rays if necessary. Challenge the class to express an equivalent angle also in degrees. Students should recognize that a 30° angle and a 390° angle are angles having the same terminal side on the coordinate plane. Define co-terminal angles. Challenge the students to represent the same angle as a negative angle i.e., angles with a clockwise rotation or -330°. Ask the class whether 5π/3 and -π/3 are co-terminal angles. Challenge the class to find equivalent angles in radians. Elicit the following generalizations from the class:
  (1) An angle θ expressed in degrees has co-terminal angles in the form θ ± 360n°, where n is an integer.
  (2) An angle θ expressed in radians has co-terminal angles in the form θ ± 2πn radians, where n is an integer.

- In order to summarize the lesson, challenge the class to give the degree measure and radian measure of the angle formed by the hour hand and the minute hand of a clock at 2:30 A.M. Elicit from the class that the hour hand is halfway between the two and the three. Students should recognize that the angle represents 7/24 of the clock, therefore, 7/24• 360° = 105° or 7π/12 radians.

Lesson plan by Stephen Burns, Tricia Weisberg, Aleksandr Pariyskiy and Nathan Alexander

Lesson 7.1: Transparency #1

WHEN AN ARC OF A CIRCLE HAS THE SAME LENGTH AS THE RADIUS OF THE CIRCLE, THE MEASURE OF THE CENTRAL ANGLE IS THE DEFINITION OF 1 RADIAN.



1 Radian is approximately 57.3°.