

TOPIC 7.2: SECTORS OF CIRCLES

PERFORMANCE OBJECTIVES

Students will be able to:

- find the arc length of a sector of a circle
- find the area of a sector of a circle
- solve problems involving apparent size
- assess the effects of offensive music on their own problem solving ability

MATERIALS

Graphing Calculator, overhead projector

STRATEGIES

- As a “do now” for this lesson, sketch a circle and a central angle of 45° and a radius of 12. Elicit from the students the (a) arc length and (b) area of the sector. Elicit that the answers for these two problems are (a) arc length = $\frac{45^\circ}{360^\circ} 2\pi(12) = 3\pi$ and (b) area of a sector = $\frac{45^\circ}{360^\circ} \pi(12)^2 = 18\pi$. In the discussion, elicit that the fraction of the area or circumference is obtained by dividing the central angle by either 360 or 2π , depending on whether the central angle is given in degrees or radians.
- For a second problem, and to use radian measure, try a circle with $\theta = 4$, $r = 2$. Elicit that the arc length and area of the sector are 8 and 8. Elicit from the students the formulas for arc length s and area K of a sector with central angle of θ . As a medial summary, place on the board these formulas. If θ is in radians: $s = r\theta$ and $K = (1/2) r^2\theta$. Propose the following problem: The apparent size of a tall building 2 km away is 0.05 radians. What is the building’s approximate height? Elicit from the students that apparent size is not a distance measure but rather an angle measure. Elicit from the students that an arc, for a small angle measure, is very nearly a straight line, and that in this case, the building is approximately 100m high.
- Pose a problem involving manipulation of the two new formulas: A sector of a circle has arc length 6 cm. and area of 75 cm^2 . Find the radius and the measure of the central angle.
- A phonograph record with diameter 12 inches turns at $33\frac{1}{3}$ revolutions per minute. How many radians does the record spin in a minute? Find the distance that a point on the rim travels in one minute. Elicit that each minute the point travels through $\theta = 33\frac{1}{3} \bullet 2\pi = \frac{200\pi}{3}$

radians. In that minute, the point travels a total of $S = r\theta = 6\left(\frac{200\pi}{3}\right) = 400\pi$ inches ≈ 105 feet.

- A CD does not spin at a constant rate, but rather the rotational speed varies depending on the position of the laser. When the laser is at the outer edge of the disk, the player spins at the slowest speed, 200-rpm. The diameter of the disk is 11.9 cm. At the slowest speed, how many degrees does the disk spin in a minute? What is the approximate distance that a point on the outer edge travels at the slowest speed in one minute? What is the speed in cm/s? How does this compare to the results with the record? Elicit that the disk spins

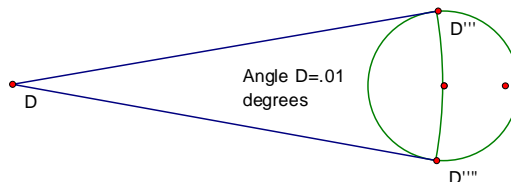
$(360^\circ)(200\text{revolutions}) = 72,000^\circ$ in one minute $= \frac{72000^\circ}{57.3^\circ}$ or 1257 radians in one minute.

Using the formula $S = \theta r$, we have $(1257 \text{ radians})\left(\frac{11.9}{2} \text{ cm.}\right) = 7479 \text{ cm.}$ as the distance this point on the outer edge of the disk travels in one minute. Dividing by 60 seconds, we have approximately **125 cm/s** as the approximate **speed of the point on the outer edge of the disk** in cm/s. Converting 7479 cm. to feet, we have $\frac{7479\text{cm.}}{2.54} = 2944$ inches, then dividing by

12 we have 245 feet. On the other hand, a point on the outer edge of a $33\frac{1}{3}$ rpm 12 inch vinyl

record rotates through $(360^\circ)\left(33\frac{1}{3} \text{ rpm}\right) = 12000^\circ = 209.4$ radians. Using the formula $S = \theta r$, we have $(209.4 \text{ radians})(6 \text{ inches}) = 1256.5$ inches of travel by the point on the outer edge of the vinyl record. This is equivalent to $(1256.5 \text{ inches})(2.54) = 3191.6$ cm. Dividing this by 60 seconds, we have **53.2 cm/s** as the **speed of the point on the outer edge of the vinyl record**. The CD travels at a speed of more than twice the vinyl.

- (Use example 2 from the text as an optional example) Jupiter has an apparent size of .01 degrees when it is 8×10^8 km from earth. Find the approximate diameter of Jupiter. Lead the class through a discussion of imagining point D (our viewpoint) as the center of the circle, Earth. The distance between our viewpoint and the circle is the radius of the sector of the arc; that is the distance from Jupiter to where we are standing. Now, you can see that the arc from points D''' to point D'''' is the arc length, that is the apparent size of Jupiter. **Using the formula 1, $s = (.01/360) * (2\pi * (8 \times 10^8)) = \text{approximately } 140,000 \text{ km.}$**



Lesson plan by Craig D. Smith, Wendy Chen, Binna Jiang and Suzanne Apicos

