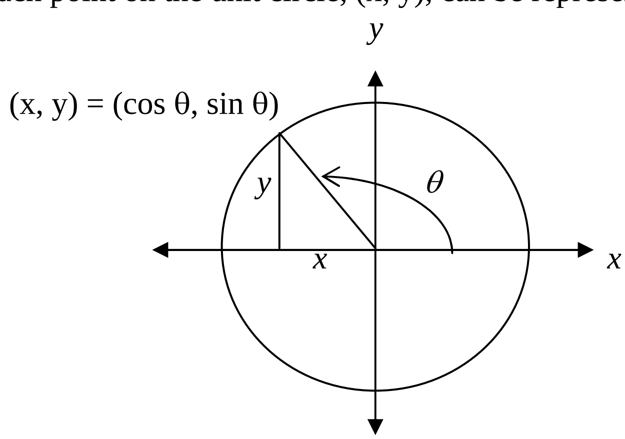
TOPIC 7.3: THE SINE AND COSINE FUNCTIONS

PERFORMANCE OBJECTIVES
Students will be able to:

- find values of sine and cosine functions
- determine the sign of the function
- solve simple trigonometric equations

STRATEGIES.

- Pose the following questions as the Do Now to get the lesson started: Draw a diagram that fits the following equation: (a) $\sin\theta = \frac{5}{13}$ (b) $\sin\theta = -\frac{5}{13}$.
- Part (a) from the Do Now can be solved using a right triangle, and the definition of the sine function, $\frac{opposite}{hypotenuse}$. For part (b) elicit from the students that a circle drawn in the coordinate plane can be used. Review that $\sin\theta = \frac{y}{r}$ and $\cos\theta = \frac{x}{r}$, where x and y are the coordinates of a point on the circle that the side of the angle intersects, and *r* is the radius of the circle. So a circle with radius 13 can be used, and the terminal side of the angle passes through the point that has coordinates (x, -5). Drawing a right reference triangle given that $\sin\theta = -\frac{5}{13}$ and using the concept of Pythagorean triples, we can find that x = -12 if the angle is in the third quadrant and x = 12 if the angle is in the fourth quadrant.
- Use the following diagram to review the idea of a unit circle. Elicit that *unit* circle has a radius of 1 unit, which can be obtained by dividing the coordinates by the value of the radius of the circle. Each point on the unit circle, (x, y), can be represented by (cos θ, sin θ).



Unit Circle (whose radius is 1)

- Elicit from the students, how a unit circle whose center is O (0, 0) when drawn in the coordinate plane can be used to solve the do now. Review that sin θ = y and cos θ = x, where (x, y) are the coordinates of a point P on the unit circle, that the ∠POA = θ, where A is any point on the x-axis.

- Review with the students the sign of the sine and cosine functions in different quadrants. Review that sine is positive in the first and second quadrants, since the y-coordinate is positive there, and cosine is positive in the first and fourth quadrants, since the x-coordinate is positive there.

- Pose the following question in order to discuss the quadrantal angles:
  Without using a calculator or table, solve each equation for all θ in radians. (This example is meant to provoke a discussion about the infinite number of solutions to a trigonometric equation.)

  (a) $\sin\theta = 1$ (b) $\cos\theta = -1$ (c) $\sin\theta = 2$ (d) $\cos\theta = \frac{\sqrt{2}}{2}$.

  In solving this group of problems, use the unit circle definition of sine and cosine to get that (a) $\theta = 90° \pm 360n$, (b) $\theta = 180° \pm 360n$, (c) does not exist because sin θ is never greater than 1 and (d) $\theta = 45° \pm 360n$ and $315°.\pm 360n$.

- As a summary ask the students to complete the following table using the unit circle

Quadrant	I	II	III	IV
sin θ	$\frac{3}{5}$	$\frac{5}{13}$	?	?
cos θ	?	?	$-\frac{24}{25}$	$\frac{15}{17}$

- Consider using the Geometer's Sketchpad to illustrate the meaning of the new definitions for sin θ and cos θ. Construct a unit circle, plot a point B on the circle and measure its coordinates. Let point A be another point on the unit circle at (1, 0). Let O be the origin of the unit circle. Construct and measure ∠AOB. Measure the sine and cosine of ∠AOB and compare these numerical values to the coordinates of point B. By doing this, the students should be able to discover that cos ∠AOB = abscissa of B and sin ∠AOB = ordinate of B.

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