TOPIC 7.4: EVALUATING AND GRAPHING SINE AND COSINE CURVES

PERFORMANCE OBJECTIVES

The student will be able to

- use the unit circle and reference angles to determine the value of the sine and cosine of any angle
- use special angles to determine the sine and cosine of angles
- know which quadrants contain positive values of sine and cosine
- determine the inter-relationship between the sine and cosine graphs

MATERIALS

Overhead projector, transparencies, and graphing calculator

STRATEGIES (This lesson may be extended to two days if necessary.)

- Give the following questions as a do-now: (a) Solve 6 sin x - 3 = 0 for all values of x from -360° to 360°. (b) Solve 2 cos x + 1 = 0 for all values of x from - π to 4π.

- Review the concepts involved in solving these problems: the 30-60-90 triangle, using references angles, and evaluating negative angles. The solutions to (a) are x = 30°, 150°, -330°, and –210°. The second problem also involves radian measure. The solutions to (b) are $x = 2\pi/3^R$, $4\pi/3^R$, $8\pi/3^R$, $10\pi/3^R$ and $-2\pi/3^R$. Review the definition and usage of reference angles, as derived from the unit circle. Elicit in the discussion that any angle can be made equivalent to a particular reference angle. Stress that the reference angle, by definition, must be acute. Use the calculator to check the solution of each of the two equations. In (a) use degree mode. In (b) use radian mode. Introduce students to the idea that radian mode will not give angles in terms of π, but rather in terms of a decimal equivalent.

- Review the mnemonic device "All Students Take Calculus" which indicates which quadrants have positive values of sine, cosine and tangent, as well as, the following diagram.

Quadrant II Sin	Quadrant I All
Quadrant III Tan	Quadrant IV Cos

- Introduce the use of the sine and cosine graph as a method of solving (a) of the do now graphically. Have them enter y = sin x and y = .5 and graph each by pressing [ZOOM], [7 Ztrig]. This will set the window from -2π to 2π on the x-axis and from –4 to 4 on the y-axis. Show that the points of intersection of the two graphs are the same as the solutions to (a) of the do now that were previously calculated. In the review of this problem, review the basic sine wave, its amplitude and its period. In addition, contrast the graph with the quadrants of positive and negatives in the previous bullet, namely that, the graph is above the x-axis in the first two quadrants and below the x-axis in the third and fourth quadrant. This is consistent with the signs of the quadrants of A-S-T-C for sine. Repeat this with (b) of the do now and the cosine graph, except reset the viewing window from $-\pi$ to 4π.

- Review the 45-45-90 and 30-60-90 triangles, using the SOHCAHTOA definitions for sine, cosine and tangent to find the exact values of sin 45°, tan 60°, etc. Summarize the discussion by deriving a table showing all values of sin theta and cos theta for θ = 0°, 30°, 45°, and 60°.

- In addition, review the sine and cosine graphs to determine quadrantal values at 0°, 90°, 180°, 270°, and 360° by giving problems such as: Solve sin x = -1 for all values of x from 0 to 360. Elicit the solution of x = 270°.

- (Optional) As a summary of the previous concepts, use the transparency included at the end of this lesson and pose the following: "The latitude of a point on Earth is the degree measure of the shortest arc from that point to the equator. At what latitude is the circumference of the circle of latitude at P half the distance around the equator?" Calling r = CP, the radius of the circle of latitude at P, and R = OP, the radius of the Earth, solve for r: $2\pi r = 5(2\pi R)$ or r = .5 R. From this, develop that $\cos \alpha = \frac{r}{R}$, so that $\alpha = 60°$. This is because of the following: Inscribe a triangle with the center at the middle of the earth (point O). Without loss of generality, put the circle of latitude in the Northern Hemisphere. Draw OP, a hypotenuse to intersect the circle of latitude at point P. Name the center of the circle of latitude point C. Draw a horizontal line OE to represent the line form the center of the earth to the equator. Drop a perpendicular line from P to the line OE. Call this intersection point D. ODPC is a rectangle, since CP = r, then OD = r. From this, develop that the cosine of angle POD = $\frac{OD}{OP}$. But $\frac{OD}{OP} = \frac{r}{R}$. From the previous discussion, r = .5R, so OD/OP = .5R/R = .5. Therefore, the cosine of $\angle$ POD = .5, so m $\angle$ POD = 60.

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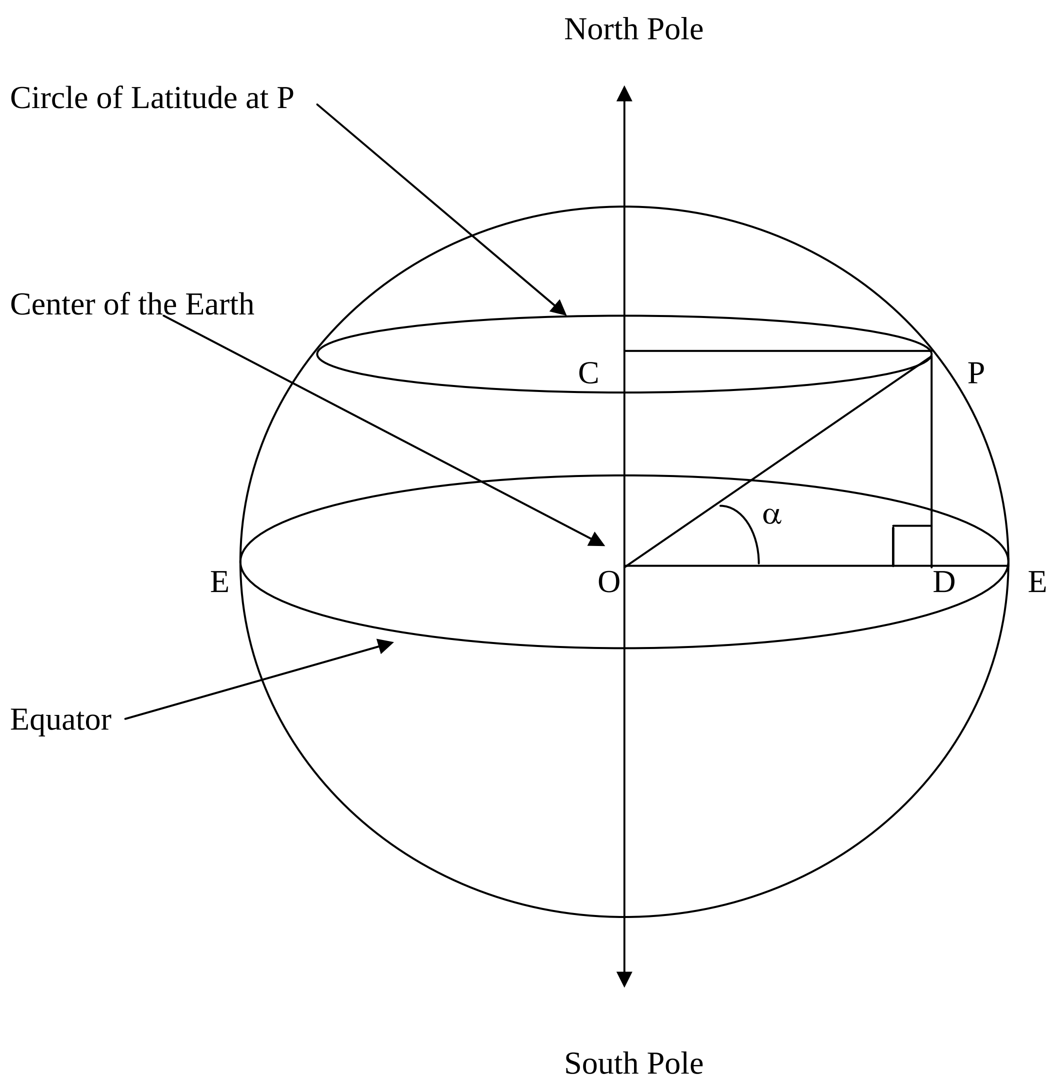
Using trigonometry to solve a geometry problem



At what latitude is the circumference of the circle of the latitude at P one half the distance around the world?