TOPIC 7.5: THE OTHER TRIGONOMETRIC FUNCTIONS

PERFORMANCE OBJECTIVES
Students will be able to:

- find the cosecant, secant or cotangent of an angle by using a calculator
- solve a trigonometric equation containing these functions
- determine the quadrants where these trigonometric ratios are positive and negative
- find the exact value of these trigonometric functions if their reference angles are 30°, 45°, or 60°
- graph the cosecant, secant and cotangent functions

MATERIALS

Graphing calculator

STRATEGIES (This lesson requires two days)

- Start the lesson with the following "Do Now":
  Find the following values on your calculator:
  (a) $\frac{1}{\tan(203°)}$ (b) cot (203°) (c) $\frac{1}{\cos(72°)}$ (d) sec (72°) (e) $\frac{1}{\sin(24°)}$ (f) csc (24°)

  Challenge the students to identify the relationship between (a) and (b), (c) and (d), and (e) and (f). From this discussion, review the reciprocal functions of y = csc x, y = sec x, and y = cot x. Formally state that csc x = $\frac{1}{\sin x}$, sec x = $\frac{1}{\cos x}$, and that cot x = $\frac{1}{\tan x}$. Also review where necessary how to find these trig values of csc x, sec x and cot x on a calculator, even though, there is no button for them. In finding the cot (203°), set the calculator into degree mode, find the tan (203°) and then find that result's reciprocal.

- Pose the following example to get the class working with these new functions: Solve for θ in the interval $0 \leq \theta \leq 360$; $2 \sec\theta - 4 = 3$. In this example, have the class solve the equation in the previously described way to get sec θ = $\frac{7}{2}$. Using the new definitions, take the reciprocal of both sides to get cos θ = $\frac{2}{7}$. Solve this equation to get the angles of 73.4° and 286.6°. When doing this example, elicit the quadrants where secant is positive (I and IV) where the cosecant is positive (I and II) and where the cotangent is positive (I and III).

- Pose this example to practice using these new trigonometric ratios in expressing the function of an angle in terms of a positive acute angle: Find the exact value of the csc (315°). Elicit that the reference angle for this angle is 45°. Also elicit that the cosecant is negative in quadrant IV. So csc (315°) = – csc (45°). Using the isosceles right triangle, elicit that – csc (45°) = $-\sqrt{2}$. Practice this with several examples whose reference angles are 30, 60, or 45. In addition, practice this with angles in radian measure, i.e., find the exact value of sin ($\frac{7\pi}{3}$)^R.

- Pose the following to the class: Use the graphing calculator to graph y = sin x and y = csc x on the same set of axes. Challenge the class to identify how to input the graph of y = csc x when there is no csc key on the calculator. Elicit that the cosecant graph would have to be input as y = $\frac{1}{\sin x}$. On a TI-83, the cosecant graph will include extraneous lines that are not part of the graph. In order to see the graph without these lines, press [MODE], [DOT], [ENTER], and a dotted version of the graph appears with no extraneous lines. Sketch the graph of y = csc x and y = sin x to see their reciprocal relationship. Elicit that when a number increases from zero to one, as does the sine graph from 0 to $\frac{\pi}{2}$ radians, its reciprocal decreases from $\frac{1}{0} or +\infty$ to 1. This is the behavior of the cosecant graph throughout the entire domain. Elicit that the domain of the cosecant function is the set of all Reals except at 0, ±π. ±2π, ±3π, etc. Elicit that the range of the cosecant graph is [1, +∞) and $[-\infty,1)$. Further elicit that there are vertical asymptotes at the points where the function is not defined.

- Have the students graph the cosine and secant functions simultaneously on the graphing calculator and again compare the two graphs. Elicit the domain as all Reals except ± $\frac{\pi}{2}$, ± $\frac{3\pi}{2}$, ± $\frac{5\pi}{2}$, etc. and range of the secant function to be the same as the cosecant function. Further elicit where the vertical asymptotes are located.

- Finally, sketch the graphs of y = tan x and y = cot x on the same set of coordinate axes. Discuss the domain, range and asymptotes for the cotangent function.

- Pose the following to practice the new trig ratios: If θ is drawn in standard position and it lies in quadrant III, and tan θ = $\frac{5}{12}$, what are the values of sin θ, cos θ, cot θ, sec θ, and csc θ.

- Summarize the lesson by asking the class to verify the Pythagorean Identity of 1 + $\cot^2\theta = \csc^2\theta$ for θ = $\frac{5\pi}{3}$ radians.

Lesson plan by Andrew Song, Harmony Fabrizio, Jennifer Lim and Sejin Chang