TOPIC 8.1: SIMPLE TRIGONOMETRIC EQUATIONS

PERFORMANCE OBJECTIVES

Students will be able to:

- solve equations involving a single trigonometric function
- use the graphing calculator to assist in solving trigonometric equations
- identify and define the inclination and slope of a line
- identify the equation of conic section, its direction angle and then sketch its curve

MATERIALS

Graphing calculator

STRATEGIES

- Start the lesson by reviewing simple trigonometric equations. Pose the following "Do Now":
  (a) Sketch the graph of y = sin x from -4π to 4π.
  (b) What is the period of this graph?
  (c) Without using the graphing calculator, when does sin $x = 0.5$?
  (d) How many solutions are there to sin $x = 0.5$?
  (e) Use the calculator to determine when sin $x = 0.6$. Find all values of x between 0 and 2π.

- The discussion of the Do Now should include the fact that the graph of y = sin x has a period of 2π, which indicates that the graph completes one cycle in 360º. Elicit from students that the number of solutions for sin x = 0.5 are infinite since the graph never ends. Two such solutions are $x = \frac{\pi}{6}$ and $x = \frac{5\pi}{6}$. Challenge the class to provide the general solution to part c, $x = \frac{\pi}{6} + 2n\pi$ and $\frac{5\pi}{6} + 2n\pi$.

- Students should be familiar with using calculators to solve sin $x = 0.6$ for x in degree mode. Use the equation to challenge the class to solve for x in radian mode. First, set the calculator to radian mode. Then, press [2nd], [SIN], [.], [6]. The answer is 0.6435^R. This provides practice for the class to work in radians not in multiples of π. Discuss strategies for finding other value(s) of x in the 0 to 2π interval. Ask, "In what quadrants is the graph of sin x positive?" Elicit that 0.6435 is only the reference angle for the other solutions. Since sin x is also positive in quadrant II, find the other answer by subtracting $\pi - 0.6435$. If students need more practice, pose the following:
  (a) Solve for x: cos $x = -0.8$, $0 \le x < 2\pi$:
  (b) Find all θ to the nearest tenth of a degree if $3\cos\theta + 9 = 7$, $0 \le \theta < 360°$

- Summarize the steps for solving a trigonometric equation:
  (1) Isolate the trigonometric ratio on one side of the equal sign.
  (2) Determine if the solution is to be in radians or degrees and set the calculator accordingly.
  (3) Find the reference angle and decide when the quadrants are positive and negative.
  (4) Using (3), find all solutions in the given interval.

- Introduce the inclination of a line as the angle ϕ (phi), where $0^\circ < \phi < 180^\circ$, measured from the positive x-axis to the line, challenge the class to find the angle that the line $y = x - 3$ makes with the positive x-axis. Elicit that, by inspection, the angle is 45°. Ask the class to what the tangent of 45° is and what the correlation is to the equation of the given line. Introduce the following theorem: For any line with slope *m* and inclination ϕ, $m = \tan \phi$ for all $\phi \neq 90^\circ$. Elicit that when the line is perpendicular to the x-axis, it is a vertical line and there is no slope.

- Apply this theorem to the following: To the nearest degree, find the inclination ϕ of the line $3x + 8y = 16$. Elicit that the slope may be easily found by rewriting the equation into $y = mx + b$ form as $y = -\frac{3}{8}x + 2$. Accordingly, $\phi = \tan^{-1}(-\frac{3}{8}) = 159^\circ$.

- In order to motivate the concept of how ϕ applies to conic sections, pose the following: Graph the following on your graphing calculator: $y = \frac{x+\sqrt{4-3x^2}}{2}$, $y = \frac{x-\sqrt{4-3x^2}}{2}$, $y = x$ and $y = -x$, all on the same set of axes. Remind students to use parenthesis around the entire numerator when entering it into the calculator. Zoom in once to get a clear picture of what these two graphs represent and elicit that they form an ellipse. Challenge the class to express this equation without radicals. Elicit that multiplying both sides of the equation by 2, transposing the x to the left side and squaring, we get $4y^2 - 4xy + x^2 = 4 - 3x^2$; this simplifies to $x^2 - xy + y^2 = 1$. Elicit that this is the equation of an ellipse whose major and minor axes have rotated through an angle of Φ°. In order to find angle Φ, we need to find the point of intersection of $y = x$ with our first equation in the quadrant I. Use the [2nd], [TRACE], [INTERSECT] feature and move the cursor near the point of intersection with the blue arrow keys, [ENTER], select the graph of $y = x$, [ENTER], then [ENTER], to get the point of intersection (1, 1). Therefore, the angle $\Phi = 45^\circ$ since the point (1,1) forms an isosceles right triangle with the line $y = x$.

- Summarize this as follows: From work that is beyond the scope of this course, we have a way to find the angle of rotation of a conic section with the positive x-axis as follows: If $Ax^2 + Bxy + Cy^2 + Dx + Ey + F = 0$, and $A = C$, $\Phi = 45^\circ$. If $A \neq C$, then $\tan 2\Phi = \frac{B}{A-C}$ where $0 < 2\Phi < \pi$. Use this to find the direction angle Φ for $x^2 - 2xy + 3y^2 = 1$. Let $\tan 2\Phi = \frac{-2}{1-3}$. So $2\Phi = \frac{\pi}{4}$ and $\Phi = \frac{\pi}{8}$.

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