TOPIC 8.2: SINE AND COSINE CURVES

PERFORMANCE OBJECTIVES

Students will be able to:

- find the amplitude and period of sine and cosine graphs
- identify that the amplitude involves vertical stretching or shrinking and while the period involves horizontal stretching or shrinking
- graph sine and cosine functions
- solve trigonometric equations algebraically and graphically

MATERIALS

Overhead projector, graphing calculator, graph paper

STRATEGIES

- As a "Do Now", ask students to find the amplitude and period of the graph y = 3 sin 2x and graph the function from $0 \le x < 2\pi$. Review that when sketching graphs of sine and cosine functions, we need the 5 critical points in one period of each graph from the: (a) intercepts, (b) maximum points and (c) minimum points. To find those 5 key points in y = a sin bx:
  (1) find the amplitude *a* and frequency *b*. The frequency tells us how many complete sine or cosine waves there are in an interval of 2π. Elicit that the amplitude of y = a sin x and y = a cos x is the largest value of y and is given by amplitude = |a|. The amplitude represents a vertical stretch or shrink of the curve.
  (2) divide the frequency, b, into 2π to get the period. The period tells us how many radians it takes for the graph to complete one complete sine wave or cosine wave.
  (3) divide the period into 4 equal parts to get the spacing of the critical points
  (4) For y = a sin bx, start at (0,0). Then, every $\frac{2\pi}{4b}$ radians, place a critical point, first at $\frac{2\pi}{4b}$ radians at the maximum point, next to zero at $\frac{2\pi}{4b}$ radians later, to the minimum point $\frac{2\pi}{4b}$ later, and finally back to zero $\frac{2\pi}{4b}$ later. Repeat, as needed, until 2π is reached. For the given y = 3 sin 2x, the critical points are spaced at $\frac{2\pi}{4 \bullet 2}$ radians at $(0, 0), (\frac{\pi}{4}, 2), (\frac{\pi}{2}, 0), (\frac{3\pi}{4}, -2), (\pi, 0), (\frac{5\pi}{4}, 2), (\frac{3\pi}{2}, 0), (\frac{7\pi}{4}, -2), (2\pi, 0)$.
  A sine wave is drawn through these points. Verify this by the graphing calculator.

- Have the students solve the following equation graphically and then algebraically.
  Solve: $3 \sin 2x = 1$ for $0 < x < 2\pi$.
  Elicit the steps needed to solve the problem using the graphing calculator.
  (1) Set calculator to radian mode.
  (2) Go to the [y=] screen and enter as $y_1 = 3 \sin 2x$ and as $y_2 = 1$.
  (3) Press [GRAPH] and notice that the graphs cross four times between 0 and 2π.
  Using the calculator, press the [CALC] button, then the [INTERSECT] button. When prompted for the first curve, move the cursor to the point of intersection and press [ENTER], and when prompted for the second curve, press [ENTER] again. The answer to the point furthest to the left is: (.17, 1) repeat the process for the other 3 points. The other answers are (1.40, 1), (3.31, 1) and (4.54, 1).
  Solving algebraically, we have $3 \sin 2x = 1$ or $\sin 2x = \frac{1}{3}$. Thus, $2x = \sin^{-1}(\frac{1}{3})$ and $2x = .34$ radians and $x = .17$ radians. In quadrant II, $2x = 2.80$ radians, so $x = 1.40$ radians. Adding $2\pi + 2x$, places another solution in the first quadrant at $2x = 6.62$ radians and thus $x = 3.31$ radians. Finally adding 6.28 (2π) to 2.80 we get an angle in the second quadrant, $2x = 9.08$ radians and $x = 4.54$. Check each solution on the calculator to confirm sin 2x is 1.

- Include an example of a cosine graph in this lesson. Ask the class to perform the same tasks as in the previous example for $y = 3 \cos \frac{1}{2}x$ from $-\pi$ to π. A standard cosine wave goes in the following pattern: (0, 1) maximum, $(\frac{\pi}{2}, 0)$ intercept, $(\pi, -1)$ minimum, $(\frac{3\pi}{2}, 0)$ intercept, and $(2\pi, 1)$ maximum. Start placing the critical points at (0, 3). The critical points in this graph are every π radians, indicating that the next point is plotted at $(\pi, 0)$. Going in the negative direction, we have $(-\pi, 0)$. Sketch a cosine wave through these three points. Verify this with the graphing calculator. In addition, elicit that if the equation is $y = -\sin x$, this would be a reflection in the x-axis and the critical point would be adjusted accordingly.

- Pose an example of a graph where the students need to write the equation of the trigonometric function given the amplitude and period. Challenge the class to find the equation of a cosine graph whose amplitude is 3 and whose period is $\frac{\pi}{2}$ and passes through the point $(\frac{\pi}{2}, -3)$. Elicit that this cosine graph has an amplitude of 3, period of $\frac{\pi}{2}$ and that there will be 4 cosine waves in a 2π interval. Use the general equation $y = a \cos bx$ to get the equation as: $y = 3 \cos 4x$. However because the graph passes through $(\frac{\pi}{2}, -3)$, the sign of the amplitude is negative, since the graph is a reflection of the sine wave in the x-axis.
  The final answer is $y = -3 \cos 4x$.

Lesson plan by Zakir Islam, Rhonda Jeffrey, David Walters, Timothy Evans and Soledad Fernandez