TOPIC 8.3: MODELING PERIODIC BEHAVIOR

PERFORMANCE OBJECTIVES

Students will be able to:

- understand the definition of period function and the terms amplitude and period
- use trigonometric functions to model periodic behaviors
- given a trigonometric function, find its' period and amplitude
- sketch a trigonometric function, showing its' period and amplitude
- modify a graph with the equation y - k = A sin B(x - h) and y - k = A cos B(x - h) from the basic form y = sin x and y = cos x, respectively

MATERIALS

Overhead projector, graphing calculator

STRATEGIES

- As a "Do Now" pose the following examples to the class to start the lesson:
  On the same set of axes, sketch the graphs of $y = 3 \sin x$ and $y = 3 \sin (\pi - x)$

- Review sketching trigonometric functions of the form of $y = a \sin bx$. A function is **periodic** if there is a positive number p, called the period of f, such that $f(x+p) = f(x)$ for all x in the domain of f. In all three cases, the period is 2π because the graphs repeat themselves every 2π radians. As stated in the definition above of period, we have $f(\theta) = f(\theta + 2\pi)$. If a period function has a maximum M and a minimum m, then the **amplitude** A is defined by $A = \frac{(M-m)}{2}$. In all three cases above, the amplitude is 3. If a period function f has period p and amplitude A, then $y = cf(x)$ has period p and amplitude cA, and $y = f(cx)$ has period $\frac{p}{x}$ and amplitude A. In the second example, we have $y = 3\sin(\pi - x)$ and it may be rewritten as $y = 3\sin[-(x - \pi)]$, a graph that is a translation of the first graph π units to the right and a reflection in the x-axis. The rules for transformations apply to these trigonometric functions.

- Review with the students about the following facts and show by stretching graphs.

If the equation y = f(x) is changed to:	***Then the graph of y = f(x) is:***
$y = -f(x)$ $y = f(-x)$ $x = f(y)$	Reflected in the x-axis Reflected in the y-axis Reflected in the line y = x
$y = cf(x),\ c > 1$ $y = cf(x),\ 0 < c < 1$	Stretched vertically Shrunk vertically
$y = f(cx),\ c > 1$ $y = f(cx),\ 0 < c < 1$	Shrunk horizontally Stretched horizontally
$y - k = f(x - h)$	Translated h units horizontally and k units vertically

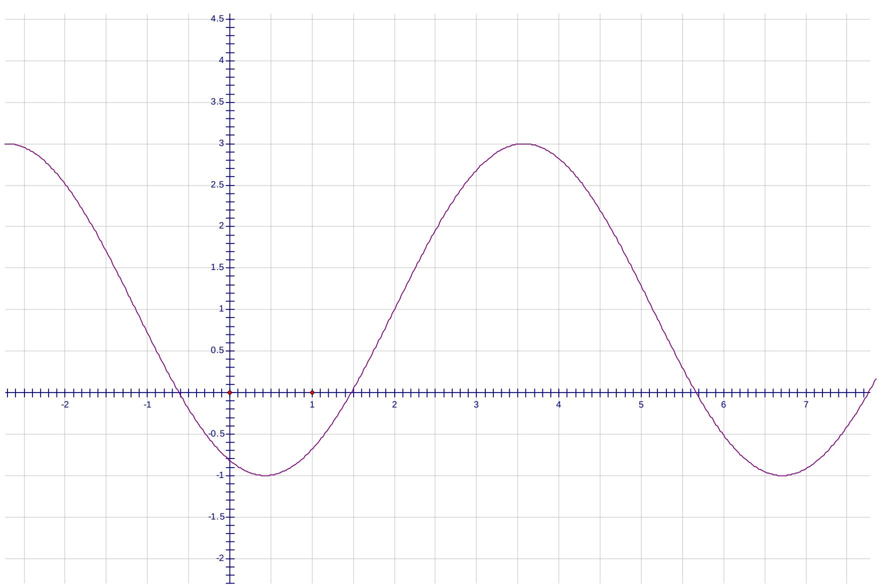
- For any sine function, we have

Function	***Period***	***Amplitude***
y = sin x	2π	1
y = A sin x	2π	A
y = sin B x, B> 0	$\frac{2\pi}{B}$	1
y = A sin B x	$\frac{2\pi}{B}$	A

- In general, the graph of (y- k) = c sin (x - h) is obtained from the graph y = c sin d x by translating h units horizontally and k units vertically. The graph of y = cos x = sin ($\frac{\pi}{2}$ - x), is obtained by reflecting (with respect to the y -axis) and translating of the sine curve.

- Sketch the graph of y - 2 = 3 sin 4 (x + π). Elicit that we should start with the basic sine curve, y = sin x, then sketch the graph y = sin 4x , which is of period $\frac{2\pi}{4}$ = $\frac{\pi}{2}$(shrinking horizontally). Then stretching vertically 3 times, we get the graph y = 3 sin 4x. Finally, by translating π units horizontally to the left and 2 units vertically, we obtain the required graph.

- Find the amplitude of the sine graph whose diagram is shown below. Next, find its period and frequency. Then, write its equation. Elicit that its amplitude may be calculated by subtracting Max value – Min value and dividing by 2, which equals 2. The period is 2π because the curve begins to repeat at about 6.28 boxes. That then means the frequency is 1. The axis of the wave is the average of the Max and Min = $\frac{3+(-1)}{2} = 1$. The translation is 2 units to the right. Therefore, the equation is y = 2 sin (x – 2) + 1.



- Summarize the lesson in the following steps:
  1. To find the amplitude and the period respectivly for a given graph of sine or cosine functions, use the formula A = $\frac{Max - Min}{2}$ and the period p, equals the horizontal distance between successive maximum and minimum values.
  2. To write the equation of the graph, (for sine or cosine) use y – k = A sin B(x- h) where we have p = $\frac{2\pi}{B}$. To find the distances of the translation, first we have to find the axis of the wave, which is the horizontal line midway between the maximum and minimum points of the curve. The equation of axis of the curve is y = $\frac{Max - Min}{2}$

  If the equation is to be in terms of sine, select a point where the curve intersects its' axis. Determine the translation distances in moving from the point (0, 0) to this point.

  If the equation is to be in terms of cosine, select a highest point on the curve. Determine the translation distances in moving from the point (0, A) to this point.

- The concept that the period of a sine or cosine function is integral may be introduced with the example: y = sin πx. Have the class graph this on their calculator and discover that the period of this graph is 2. Explore that a frequency of π means that there are approximately 3.14 complete sine waves in a 2π interval. Further practice may be done on y = -2 cos 2πx.

- Provide the following problem below in oceanography:
  The depth of water at the end of a pier varies with the tides throughout the day. Today the first high tide occurs at 3:00 AM with a depth of 4.0 m, and the first low tide occurs at 9:24 AM with depth of 1.8 m. (a) Find a trigonometric equation that models the depth of the water t hours after midnight. (b) Find the depth of the water at noon. (c) A large boat needs at least 3m of water to moor at the end of the pier. During what time period after noon can it safely moor?

  Elicit that the amplitude of this periodic function is $\frac{4.0 - 1.8}{2}$ = 1.1. The period is twice the difference between low tide and high tide with the time converted to fractions of an hour 2(9.4-3) = 12.8 = $\frac{2\pi}{B}$. So B = .491. The axis of the wave is the average of the low and high heights and is $\frac{4.0 + 1.8}{2}$ = 2.9. An equation that models the depth D at time *t* hours after midnight is: D – 2.9 = 1.1 cos [.491(t – 3)]. To answer the (b) part of the question, substitute 12 into the equation for *t* to get 2.582 m. To answer the (c) part of the question, enter the graph for D = 1.1 cos [.491(t – 3)] + 2.9 and D = 3 and solve simultaneously. Elicit that the two curves intersect at t = 6.0, t = 12.8, and finally at t = 18.8. After noon, the boat may safely dock between t = 12.8 (12:48 PM) and t = 18.8 (6:48 PM).

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<u>Modeling Periodic Behavior</u>

Identify the amplitude, frequency and period of each:

$$y = 3\sin \pi x$$

$$y = 2 \cos 2\pi(x - 4)$$

Use this to set up and solve the following:

The depth of water at the end of a pier varies with the tides throughout the day. Today the first high tide occurs at 3:00AM with a depth of 4.0 meters and the first low tide occurs at 9:24 AM with a depth of 1.8 meters.

(a) Find a trigonometric equation that models the depth of the water t hours after midnight.

(b) Find the depth of the water at noon.

(c) A large boat needs at least 3 meters of water to moor at the end of the pier. During what time period can it safely moor?

Use the following to write the equation:

f(t) = y = (amplitude)[sin or cos](frequency)(x - horizontal shift) + (vertical shift)