TOPIC 8.5: SOLVING MORE DIFFICULT TRIGONOMETRIC EQUATIONS

PERFORMANCE OBJECTIVES

Students will be able to:

- solve trigonometric equations using factoring, the quadratic equation and substituting with trigonometric identity
- solve trigonometric equations on graphing calculators

MATERIALS

Graphing calculator

STRATEGIES

- Pose the following "Do Now" to start the lesson: Ask students to compare, contrast and then solve the following two equations and answer the question following:
  (1) $x^2 - 3x - 4 = 0$ (2) $\cos^2\theta - 3\cos\theta - 4 = 0$
  (3) How many solutions does each equation have?

- The procedure for solving a quadratic equation may be quickly reviewed.
  In solving (1), using factoring and the zero property rule, we have $(x + 1)(x - 4) = 0$ with $x = -1$ or $x = 4$.
  In solving (2), $\cos^2\theta - 3\cos\theta - 4 = 0$, we have $(\cos\theta + 1)(\cos\theta - 4) = 0$ with $\cos\theta = -1$ or $\cos\theta = 4$.
  Elicit that a trigonometric equation requires one additional step, namely solving for θ.
  For this, we can use the calculator to determine that $\theta = \cos^{-1}(-1)$ or $\theta = \cos^{-1}(4)$.
  Therefore for $\theta = \cos^{-1}(-1)$, $\theta = 180^o \pm 2n\pi$ ($n \varepsilon$ integers). Since the range of the cosine function is all real numbers between –1 and 1, $\cos\theta = 4$ has no solution.

- Have students solve the following example that shows that a trigonometric equation should be expressed in one trigonometric variable before solving:
  Solve: $\sin^2 x - \sin x = \cos^2 x$ for $0 \leq x < 2\pi$.

  Elicit that there are two trigonometric variables, sin x and cos x in this equation. We need to use the Pythagorean Identity to eliminate $\cos^2 x$ from the right side. Since $\cos^2 x = 1 - \sin^2 x$, the new equation becomes $\sin^2 x - \sin x = 1 - \sin^2 x$. Simplifying we have $2\sin^2 x - \sin x - 1 = 0$. Factoring we have $(2\sin x + 1)(\sin x - 1) = 0$.
  Setting each factor equal to zero we have $\sin x = -\frac{1}{2}$ or $\sin x = 1$. The first equation gives us a reference angle of $\frac{\pi}{6}$. Since sin x is negative, x must be in the third and fourth quadrants, so $x = \frac{7\pi}{6}$ radians, and $\frac{11\pi}{6}$ radians.

For the second equation, we have x = $\frac{\pi}{2}$ radians which students need to recognize is a quadrantal angle and is not put in quadrants. Students should also become familiar with the approximate decimal value of the three solutions, 3.67 radians, 5.76 radians and 1.57 radians.

- Have the class solve the following equations involving several trigonometric variables:
  (1) Solve for x: sin x tan x = 3 sin x, $0 \leq x < 2\pi$, to the nearest tenth of a radian.
  Solution: Write sin x tan x = 3 sin x as sin x tan x – 3 sin x = 0. Then factor and get, sin x (tan x – 3) = 0. Then sin x = 0 or tan x = 3. For sin x = 0, x = 0 radians and 3.14 radians; for tan x – 3, x = 1.25 radians and 4.39 radians

  (2) Solve for θ: 2 sin θ = cos θ, $0^o \leq \theta < 360^o$ to the nearest tenth of a degree.
  Solution: 2 sin θ = cos θ. Dividing both sides by cos θ, we have 2 tan θ = 1,

  tan θ = $\frac{1}{2}$, and θ = 26.6° and 206.6°

- Summarize the lesson by having the students solve the following equation two ways, algebraically and using the graphing calculator.
  Solve for θ: 2 sin θ = cos θ + 1, $0 \leq \theta < 2\pi$ to the nearest hundredth of a radian.

  We may trying to solve by getting the equation into one trigonometric variable by using the Pythagorean Identity $\sin^2 \theta = 1 - \cos^2 \theta$, thus $\sin \theta = \pm \sqrt{1-\cos^2 \theta}$

  $2 (\sqrt{1-\cos^2 \theta}) = \cos \theta + 1$. Squaring both sides we get

  $4 (1 - \cos^2 \theta) = \cos^2 \theta + 2 \cos \theta + 1$. Using the distributive property on the left side and combining like terms, we have

  $5 \cos^2 \theta + 2 \cos \theta - 3 = 0$. Factoring we have

  $(5 \cos \theta - 3) (\cos \theta + 1) = 0$. Setting each factor equal to zero we have:

  $\cos \theta = \frac{3}{5}$ or $\cos \theta = -1$

  θ = .93 radians and 5.35 radians, θ = 3.14 radians

In any radical equation where both sides were squared, we must check for extraneous roots:

2 sin .93 = cos .93 +1	2 sin 5.35 = cos 5.35 + 1	2 sin 3.14 = cos 3.14 + 1
1.6 = 1.6	$-1.6 \neq 1.6$	0 = 0
	REJECT	

The solutions are .93 radians and 3.14 radians

Graphic Solution: Since we need to find the roots of the equation, we must have the intersection point(s). In order to see this on a graphing calculator we need to write the original equation as two separate equations Y_1 = 2 sin x and Y_2 = cos x + 1. Have the students enter the equations in their calculators and then find the points of intersections. They should conclude that the solutions are approximately .93 radians and 3.14.

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