THE STATE EDUCATION DEPARTMENT / THE UNIVERSITY OF THE STATE OF NEW YORK / ALBANY, NY 12234
Curriculum, Instruction, and Instructional Technology Team - Room 320 EB
www.emsc.nysed.gov/ciai
email: emscnysmath@mail.nysed.gov

Algebra 2 and Trigonometry

Sample Tasks for Integrated Algebra, developed by New York State teachers, are clarifications, further explaining the language and intent of the associated Performance Indicators. These tasks are not test items, nor are they meant for students' use.
Note: There are no Sample Tasks for the Geometry Strand. Although there are no Performance Indicators for this strand in this section of the core curriculum, this strand is still part of instruction within the other strands as an ongoing continuum and building process of mathematical knowledge for all students.

Strands

Process	Content
Problem Solving	Number Sense and Operations
Reasoning and Proof	Algebra
Communication	Geometry
Connections	Measurement
Representation	Statistics and Probability

Problem Solving Strand

Students willbuild new mathematical knowledge through problem solving.

A2.PS.1 Use a variety of problem solving strategies to understand new mathematical content

A2.PS.1a
For each of the following:
Sketch a graph of the function.
Set the function equal to 0 and solve.
Find the discriminant.
What connections can you make between the discriminant, the solution(s), and the graph of the function?

$y = x^2 - x - 6$

$y = x^2 - 2x - 15$

$y = x^2 - 2$

$y = 3x^2 - 5x - 6$

$y = x^2 - 8x + 16$

$y = x^2 + 2x + 1$

A2.PS.2 Recognize and understand equivalent representations of a problem situation or a mathematical concept

A2.PS.2a
Given the following equations, determine the amplitude, period, frequency, and phase shift of each equation.

$$y = 2\sin\left(\frac{\pi}{3}(x-2)\right) - 4$$

$$y = -4 + 2\cos\left(\frac{\pi}{3}(x-3.5)\right)$$

Two students, Anthony and Chris, can be overheard discussing these equations. Anthony is certain these equations are equivalent, while Chris insists that they are different. Which student is correct? Explain your answer fully with graphs, tables, and a carefully written paragraph supporting your position.

Students will solve problems that arise in mathematics and in other contexts.

A2.PS.3 Observe and explain patterns to formulate generalizations and conjectures

A2.PS.3a
Simplify each of the following:

i^1	i^5
i^2	i^6
i^3	i^7
i^4	i^8

Can you see a pattern? If so, what conjecture can you make about the powers of i **?**

Based on your conjecture, simplify the following:

i^{14} i^{27} i^{301}

A2.PS.3b
On the same set of axes, use a graphing package or graphics calculator to graph the following functions:
$y = 2^x$, $y = 10^x$, $y = 1.3^x$, $y = e^x$

The functions above are all members of the family $y = b^x$.

What effect does changing b values have on the shape of the graph?
What is the *y*-intercept of each graph?
What is the horizontal asymptote of each graph?

A2.PS.3c
Sketch one cycle of each of the following equations. Carefully label each graph.

$y = \sec(x)$

$y = \csc(x)$

$y = \tan(x)$

$y = \cot(x)$

Carefully graph $y = \sec(x)$ and $y = \csc(x)$ at the same time on your calculator with a window of $0 \le x \le 2\pi,\ -4 \le y \le 4$. What conclusions can you make? Precisely describe the similarities between the 2 functions.

Now, carefully graph $y = \tan(x)$ and $y = \cot(x)$ at the same time on your calculator with a window of $0 \le x \le 2\pi,\ -4 \le y \le 4$. What conclusions can you make? Precisely describe the similarities between the 2 functions.

A2.PS.4 Use multiple representations to represent and explain problem situations (e.g., verbally, numerically, algebraically, graphically)

A2.PS.4a
For each of the following values of the discriminant, state the number of x-intercepts the graph would have, and sketch a graph of a parabola that would satisfy these conditions:

144

-6

27

0

Is it possible to find more than one graph with each discriminant? If so, describe how you obtained each graph and how each discriminant affects the graph.

Students will apply and adapt a variety of appropriate strategies to solve problems.

A2.PS.5 Choose an effective approach to solve a problem from a variety of strategies (numeric, graphic, algebraic)

A2.PS.5a
A team of biologists have discovered a new creature in the rain forest. They note the temperature of the animal appears to vary sinusoidally over time. A maximum temperature of 125° occurs 15 minutes after they start their examination. A minimum temperature of 99° occurs 28 minutes later. The team would like to find a way to predict the animal's temperature over time in minutes. Your task is to help them by creating a graph of one full period, an equation of temperature as a function over time in minutes, and a table of maximum, minimum, and average temperatures for the first 3 hours.

Discuss the advantages and disadvantages of each representation.

A2.PS.6 Use a variety of strategies to extend solution methods to other problems

A2.PS.6a
Use the strategies learned in solving quadratic equations to solve the following equations. Express any irrational solutions in simplest radical form.

$$x^4 = 13x^2 - 36$$
$$t^5 - 10t^3 + 21t = 0$$
$$(m+8)(2m-3)(m+5) = 0$$
$$\left(x^2 + 5x - 7\right)\left(x + 2\right) = 0$$

A2.PS.7 Work in collaboration with others to propose, critique, evaluate, and value alternative approaches to problem solving

A2.PS.7a
With a partner, research to find three situations that can be modeled by exponential equations. Write a description of each situation and write a problem based on each of the three situations. Solve each of the problems in a different way and be prepared to present the solutions to the class.

Students will monitor and reflect on the process of mathematical problem solving.

A2.PS.8 Determine information required to solve a problem, choose methods for obtaining the information, and define parameters for acceptable solutions

A2.PS.8a
Brianna decided to invest her $500 tax refund rather than spending it. She found a bank account that would pay her 4% interest compounded quarterly. If she deposits the entire $500 and does not deposit or withdraw any other amount, how long will it take her to double her money in the account?

A2.PS.8b
Dante has to travel from Cambridge, NY to Buffalo, NY, a distance of approximately 333 miles. He estimates that he can average 20 mph faster during the 273 miles that he will be driving on the New York State Thruway than he can when he drives on other roads. If he wants to complete the trip in eight hours, find, to the nearest integer, the rate that Dante must travel on the Thruway.

A2.PS.9 Interpret solutions within the given constraints of a problem

A2.PS.9a
Last year's senior class spent $23.95 for each prom favor. This year's prom committee knows that their prom favor must be within $5.50 of last year's favor. Write an absolute value inequality that could be used to model the acceptable price range for this year's prom favor, and then solve the inequality to find the range of acceptable prices for a favor. Explain you answer.

A2.PS.10 Evaluate the relative efficiency of different representations and solution methods of a problem

A2.S.PS.10a
A bag contains three chocolate, four sugar, and five lemon cookies. Greg takes two cookies from the bag for a snack. Find the probability that Greg did not take two chocolate cookies from the bag. Explain why using the complement of the event of not choosing two chocolate cookies might be an easier approach to solving this problem.

Back to top

Reasoning and Proof Strand

Students will recognize reasoning and proof as fundamental aspects of mathematics.

A2.RP.1 Support mathematical ideas using a variety of strategies

A2.RP.1a
Read through the experiment described below. Before beginning this experiment, think about what is happening. What type of function will you expect to see? Why would you expect to see this type of function to model the data?

Based on your answer, count the number of coins that are in the cup initially and write an equation that could be used to model the number of coins remaining each time. Explain how you determined this equation. Use this equation to predict the number of times the experiment would have to be repeated until there is one coin remaining.

The Experiment

Take a handful of coins and put them into a cup. Shake the cup and pour the coins onto the desk. Count the total number of coins, and record this number.
Remove all of the coins that are face up and record the total number left.
Using the new total of coins each time, repeat the procedure until there are no coins left. When the number of coins reaches zero, the experiment is over and you should not use zero as part of your data.
Write an appropriate regression equation to model your data. What do the variables represent?
Compare this equation to the one that you wrote before the experiment began. Explain any differences between the two equations and any errors that you made in the equation that you wrote.

How many trials did it actually take until there was one coin remaining? Compare the actual number to the number predicted by the regression equation and the equation that you wrote.

Students will make and investigate mathematical conjectures.

A2.RP.2 Investigate and evaluate conjectures in mathematical terms, using mathematical strategies to reach a conclusion

A2.RP.2a
Lauren and Diana disagree about one of the rules for simplifying logarithms.

Lauren says that $\log A + \log B = \log(A + B)$ because you can factor out the log.

Diana says that $\log A + \log B = \log(AB)$, because you add exponents when you are multiplying.

Which student is correct?
Explain your answer using two different strategies such as a table, graph, algebraic proof, etc.

A2.RP.3 Evaluate conjectures and recognize when an estimate or approximation is more appropriate than an exact answer

A2.RP.3a
Based on census data, the U.S. Census Bureau has projected the population of the United States until 2050. The table below contains these predictions.

Year	2000	2010	2020	2030	2040	2050
Population (in thousands)	282,125	308,936	335,805	363,584	391,946	419,854

Make a scatter plot of the data. Why do you think that the U.S. Census Bureau expressed the population in thousands?
Determine a regression equation that could be used to model the data. Use this equation to determine the number of people expected in the United States this year.

How should you round your answer? Why?
Research the current population of the United States. How does your estimate compare to the actual population? Explain why your answer is different from the actual population.

A2.RP.4 Recognize when an approximation is more appropriate than an exact answer

A2.RP.4a
Fill in the blanks in the following chart.

Trigonometric Function	Exact Value	Approximate Value
$\sin \pi$		
$\tan 45^\circ$		
$\cos 270^\circ$		
$\sin \frac{\pi}{3}$		
$\cos$ ___	$\frac{\sqrt{3}}{2}$	
$\tan$ ___	$\sqrt{3}$	
$\cos$ ___	$\frac{\sqrt{2}}{2}$	

Under what circumstances would you use an approximation for each of these values, rather than giving an exact answer?

A2.RP.4b
Give an example of an experiment where it is appropriate to use a normal distribution as an approximation for a binomial probability. Explain why in this example an approximation of the probability is a better approach than finding the exact probability.

Students will develop and evaluate mathematical arguments and proofs.

A2.RP.5 Develop, verify, and explain an argument, using appropriate mathematical ideas and language

A2.RP.5a
What is the range of the function $f(x) = \sin x$? Based on your answer, what is the range of the function $f(x) = \csc x$? Explain your answer.

A2.RP.6 Construct logical arguments that verify claims or counterexamples that refute claims

A2.RP.6a
Find a counterexample to refute each of the following claims:

All functions of the form $f(x) = x^n$ are one-to-one.

All one-to-one functions are onto.

A2.RP.7 Present correct mathematical arguments in a variety of forms

A2.RP.7a
Demonstrate that $f(x) = 2x - 3$ and $g(x) = 0.5x + 1.5$ are inverses using at least two different strategies (numeric, graphic, algebraic).

A2.RP.7b
Starting with $\cos^2 x + \sin^2 x = 1$, and using your knowledge of the quotient and reciprocal identities, derive an equivalent identity in terms of $\tan x$ and $\sec x$. Show all your work.

A2.RP.8 Evaluate written arguments for validity

A2.RP.8a
Liza was absent from school and emailed two of her friends to help her understand how to decide if a relation is a function.
Mike said "Make a table, and see if you get two of the same *y*-values."
John said "Look at the graph. See if a vertical line crosses the graph in more than one place. If it does, then we have a function."

Which student is correct? Why? Provide a counterexample to explain any errors made by either Mike or John.

Students will select and use various types of reasoning and methods of proof.

A2.RP.9 Support an argument by using a systematic approach to test more than one case

A2.RP.9a
Sketch right triangle *LMD*, $m\angle D = 90,\ m\angle L = 45.$ Write your favorite number as the length of one of the sides. Using this information, find the lengths of the other two sides. Write the lengths as *exact* lengths. Do not use decimal approximations. Express the value of each of the following as the ratio of the sides of the triangle.

$\sin L$

$\cos L$

$\tan L$

$\sec L$

$\csc L$

$\cot L$

Reduce all of the fractions to lowest terms and compare your answers to another student's answers. What pattern do you see? What conclusions can you reach?

Fill in another number as the length of one of the sides. Compare your answers again. Based on what you have found, determine the exact values of each of the following:

$\sin 45^\circ$

$\cos 45^\circ$

$\tan 45^\circ$

$\sec 45^\circ$

$\csc 45^\circ$

$\cot 45^\circ$

A2.RP.10 Devise ways to verify results, using counterexamples and informal indirect proof

A2.RP.10a
Jenna's teacher has asked the class to find the exact value of sin(105°). Jenna's work is as follows.

$$\sin\left(105^\circ\right) = \sin\left(60^\circ + 45^\circ\right)$$

$$\sin\left(60^\circ + 45^\circ\right) = \sin\left(60^\circ\right) + \sin\left(45^\circ\right)$$

$$\sin\left(60^\circ\right) = \frac{\sqrt{3}}{2},\ \sin\left(45^\circ\right) = \frac{\sqrt{2}}{2}$$

$$\sin\left(105^\circ\right) = \frac{\sqrt{3}}{2} + \frac{\sqrt{2}}{2} = \frac{\sqrt{3}+\sqrt{2}}{2}$$

Jenna's teacher has marked this as incorrect. Using counterexamples or an indirect proof, demonstrate why Jenna's work is not correct.

A2.RP.11 Extend specific results to more general cases

A2.RP.11a
Let $P(x, y)$ be a point in quadrant one on the unit circle, $x^2 + y^2 = 1$. Draw the line segment OP. Let θ be the angle formed by $\overline{OP}$ and the positive portion of the *x*-axis. Now draw the perpendicular from *P* to meet the *x*-axis at point *M*.

State the ratio of $\frac{MP}{OP}$ in terms of θ.

State the ratio of $\frac{OM}{OP}$ in terms of θ.

State the coordinates of point *P* in terms of θ.

Substitute your coordinates into the unit circle equation to verify one of the Pythagorean identities.

Now choose *P* in a different quadrant and repeat the process. Does the identity continue to be true?

A2.RP.11c
Sketch a scatter plot whose regression model could be logarithmic and one whose regression model could be exponential. Use your knowledge of exponential and logarithmic functions to explain the differences and similarities in the two scatter plots.

A2.RP.12 Apply inductive reasoning in making and supporting mathematical conjectures

A2.RP.12a
Given the sequence $-4,\ 0,\ 4,\ 8,\ 12\ldots$, Paul notices a pattern and finds a formula he believes will find the sum of the first *n* terms. His formula is $\sum_{i=1}^{n} 4n - 8 = 2n^2 - 6n$. Show that Paul's formula is correct.

Back to top

Communication Strand

A2.CM.1 Communicate verbally and in writing a correct, complete, coherent, and clear design (outline) and explanation for the steps used in solving a problem

A2.CM.1a
One of the students in class was absent the day the class learned the technique of completing the square. Using the technique of completing the square, write an explanation of how to solve the following equation that you could give to the student who had been absent:

$$x^2 + 8x - 3 = 0$$

A2.CM.1b
Simplify each of the following. Assuming your friend is absent; write him a complete description on the steps necessary to simplify these problems. Include as much information as possible.

$\sqrt{-7}$

$\sqrt{-80}$

$\sqrt{-2a^2b^5}$

$-6\sqrt{-32x^3}$

A2.CM.2 **Use mathematical representations to communicate with appropriate accuracy, including numerical tables, formulas, functions, equations, charts, graphs, and diagrams**

A2.CM.2a
The Art Club has purchased flat sheets of cardboard to make storage boxes for the club's art supplies. They will make boxes by cutting a square from each corner of the 24 inch by 36 inch sheet of cardboard.

Draw a diagram to illustrate the given information.

Express the volume of the box as a function of the side of the square cut from the cardboard.

Make a table for the function. Use the table to find the volume of a box formed by removing a square which has a 10 inch side, and the length of the side of a square that would produce a volume of 1792 cubic inches.

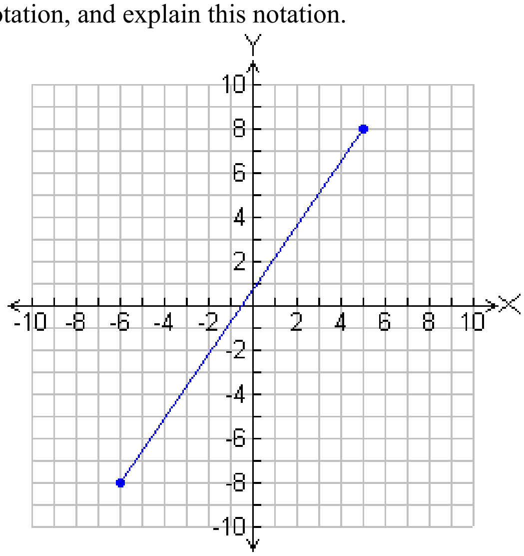
Sketch a graph of the function, and use the graph to find the side of the square that would produce the maximum volume. Find the maximum volume of the box that can be made.

Make a chart containing the side of the square removed, length, width, height and volume of the box created that the club could use for quick reference to make boxes of appropriate size for the supplies. Be sure to determine an appropriate degree of accuracy for the entries in the chart.

Students will communicate their mathematical thinking coherently and clearly to peers, teachers, and others.

A2.CM.3 **Present organized mathematical ideas with the use of appropriate standard notations, including the use of symbols and other representations when sharing an idea in verbal and written form**

A2.CM.3a
What is the domain and range of the function shown below? Express your answer in standard mathematical notation, and explain this notation.



A2.CM.4 **Explain relationships among different representations of a problem**

A2.CM.4a
Convert the equation $x^2 + y^2 + 2x - 4y - 11 = 0$ into center-radius form. When is this form of the equation more useful?

Explain how to convert from center-radius form to standard form.

A2.CM.5 Communicate logical arguments clearly, showing why a result makes sense and why the reasoning is valid

A2.CM.5a
Sally's math teacher said you could use the conjugate of a complex number to rationalize the denominator of a fraction which contains a complex number. Sally is trying to rationalize the denominator of $\frac{4+i}{3+2i}$ but cannot remember if $-3-2i$ or $3-2i$ is the conjugate. Which one is the conjugate and why?

A2.CM.5b
A group of eight students decided that they wanted to lose weight. Four of them decided to walk a mile each school day before school. The other four of them decided to walk a mile each school day after school. All eight weigh themselves each Wednesday and report their weight to their math teacher, who is keeping it confidential.

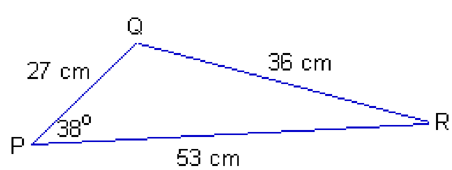
One student in the class says this is an experiment. A second student disagrees and says this is an observational study. A third student thinks this is just a survey.

After discussing the information with your partner, write a paragraph to explain why you believe the study is an experiment, an observational study, or a survey. Be clear and concise.

A2.CM.6 Support or reject arguments or questions raised by others about the correctness of mathematical work

A2.CM.6a
Two students in Ms. Baum's class were working on a problem and got different solutions. The students each were certain they had the correct solution. Who had the correct answer? Write a detailed paragraph to explain whose work was correct and why.

Given triangle PQR, find the largest angle to the nearest degree.



Student #1

$$\frac{\sin(38°)}{36} = \frac{\sin(\angle R)}{27}$$

$$36\sin(\angle R) = 27\sin(38°)$$

$$m\angle R = \sin^{-1}\left(\frac{27\sin(38°)}{36}\right)$$

$$m\angle R = 27.5$$

$$m\angle Q = 180-38-27.5$$

$$m\angle Q = 115°$$

Student #2

$$\frac{\sin(38°)}{36} = \frac{\sin(\angle Q)}{53}$$

$$36\sin(\angle Q) = 53\sin(38°)$$

$$m\angle Q = \sin^{-1}\left(\frac{53\sin(38°)}{36}\right)$$

$$m\angle Q = 65°$$

A2.CM.6b

It has been decided that 78 people need to be surveyed to decide the public's opinion on a school building project. A student suggests that they survey the first 78 people who enter the school.

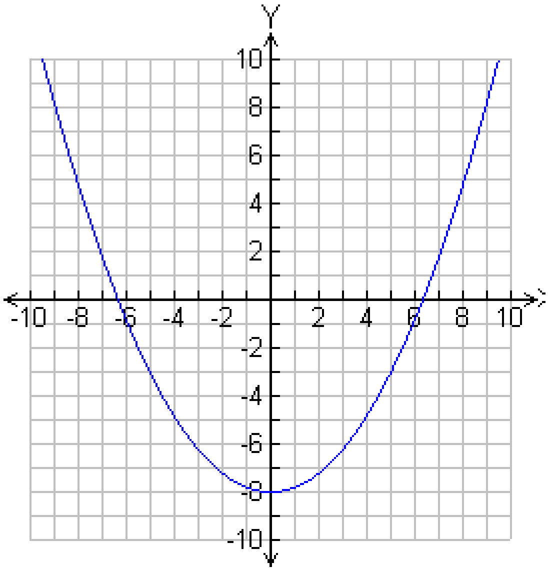
> Do you think that this proposed way of sampling is an unbiased way to perform the survey or can you describe a better way to achieve a fair and accurate response?

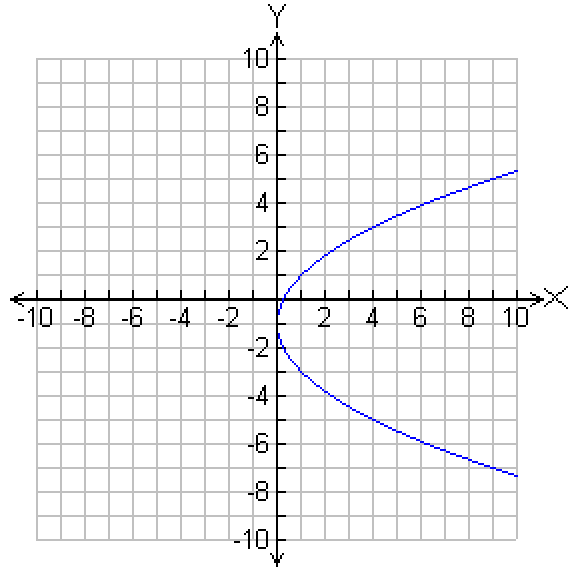
Students will analyze and evaluate the mathematical thinking and strategies of others.

A2.CM.7 Read and listen for logical understanding of mathematical thinking shared by other students

A2.CM.7a

Which of the following two graphs represents a function? Write an explanation of the reasoning used in obtaining this answer. Read the explanation provided by another student, and compare solutions.





A2.CM.8 Reflect on strategies of others in relation to one's own strategy

A2.CM.8a

Factor each of the expressions completely. Compare your answers with others in the class, discussing the process you followed to obtain the answers.

$4s^4 - 64$ $\quad$ $a^8 - b^4$

$3x^2 - 6x - 24$ $\quad$ $5y^3 + 15y^2 - 20y$

A2.CM.9 Formulate mathematical questions that elicit, extend, or challenge strategies, solutions, and/or conjectures of others

A2.CM.9a

Describe a situation that would require calculating a permutation, and another situation that would require calculating a combination. Share your thoughts with a classmate and discuss the strategies used to determine when to calculate a permutation and when to calculate a combination. Write a brief summary of your conclusions and be prepared to discuss your ideas with the class.

Students will use the language of mathematics to express mathematical ideas precisely.

A2.CM.10 Use correct mathematical language in developing mathematical questions that elicit, extend, or challenge other students' conjectures

A2.CM.10a
Solve each quadratic equation and use the roots to complete the table:

Quadratic Equation	**Roots**	**Sum of Roots**	**Product of Roots**
$x^2 - 9 = 0$			
$x^2 - 2x - 8 = 0$			
$x^2 - 16x + 64 = 0$			
$2x^2 + 5x + 3 = 0$			
$6x^2 - 7x - 5 = 0$			
$x^2 - 9x = 0$			

Use the information in the table to make a conjecture about the relationship between a quadratic equation and the sum and product of its roots. Use correct mathematical language to write your conjecture.
When finished writing, exchange the paper with another student. Read the conjecture and decide whether the conjecture is valid.
If it is valid, write questions that will help the student prove that the conjecture is valid.
If the conjecture is invalid, write questions that challenge the conjecture based upon correct mathematics and mathematical reasoning.
Return the paper. Respond to the student's questions using correct mathematical language and reasoning to prove or disprove the validity of the conjecture.

A2.CM.11 Represent word problems using standard mathematical notation

A2.CM.11a
Maya has decided to train for a marathon (26 miles) and decides to set up a practice schedule to build her stamina. When she begins, she can only run 3 miles, but she intends to train every day and increase her run by 2 miles each week Find a pattern and write a formula that will give the number of miles Maya can run in week *n*.

Using the formula, how many weeks will Maya need to train in order to be ready for the marathon?

A2.CM.12 Understand and use appropriate language, representations, and terminology when describing objects, relationships, mathematical solutions, and rationale

A2.CM.12a
Write a word problem that illustrates an application of the Law of Sines, Law of Cosines, or a combination of both. Be creative, but be certain to use appropriate language and mathematical terminology when describing the problem situation. Also prepare a complete solution of the problem including a correctly labeled diagram and full mathematical explanation of how to solve the problem.

A2.CM.13 Draw conclusions about mathematical ideas through decoding, comprehension, and interpretation of mathematical visuals, symbols, and technical writing

A2.CM.13a
Research the topic of "The Set of Complex Numbers" to answer the following:
Define: complex number, imaginary number, real number, imaginary part, real part, conjugate

When speaking of a complex number what do *a, b,* and *i* represent or mean?

Give an example and draw a graphical representation of a complex number that fits each description:

$a \neq 0$ and $b \neq 0$
$a = 0$ and $b \neq 0$
$a \neq 0$ and $b = 0$

Write the conjugate of each of the complex numbers, and draw the graphical representation of each conjugate.

How are the graphical representations of real numbers and complex numbers the same and how are they different?

How are the graphical representations of a complex number and its conjugate the same and different?

Back to top

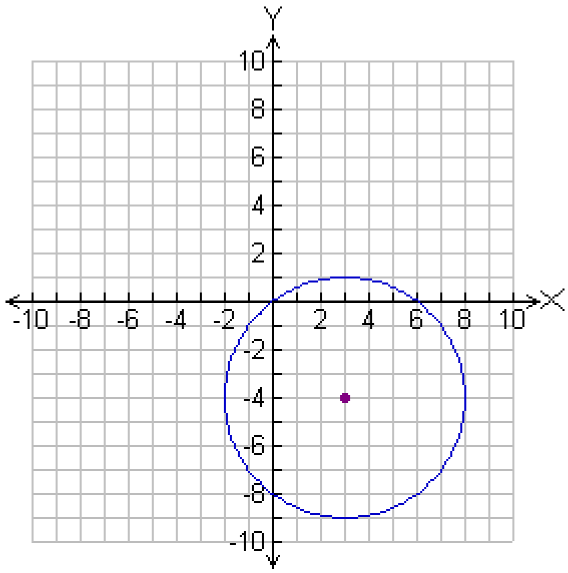
Connections Strand

Students will recognize and use connections among mathematical ideas.

A2.CN.1 Understand and make connections among multiple representations of the same mathematical idea

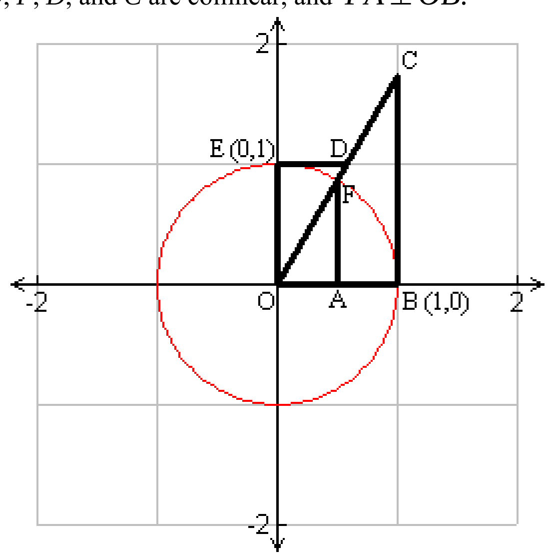
A2.CN.1a

Describe the circle whose graph is shown below. Write an equation for the circle and name six points that lie on the circle.



A2.CN.1b

In the accompanying diagram, unit circle *O* has radii $\overline{OB}$, $\overline{OE}$, and $\overline{OF}$, $\overline{CB}$ is tangent to circle *O* at *B*, and $\overline{ED}$ is tangent to circle *O* at *E*. Points *O*, *F*, *D*, and *C* are collinear, and $\overline{FA} \perp \overline{OB}$.



If $m\angle COB = \theta,$ name the line segment whose measure is each of the following:

$\sin\theta$

$\cos\theta$

$\tan\theta$

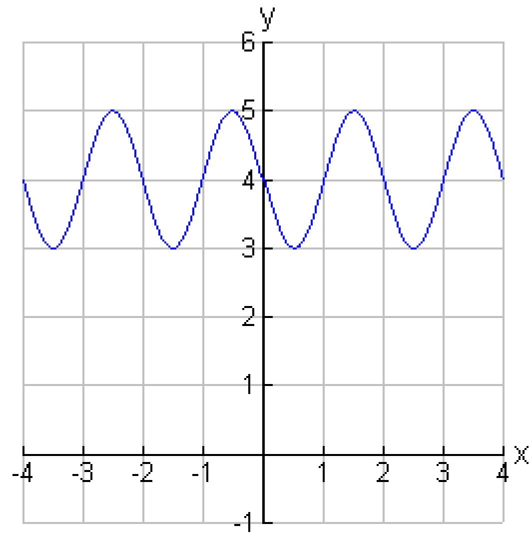
$\sec\theta$

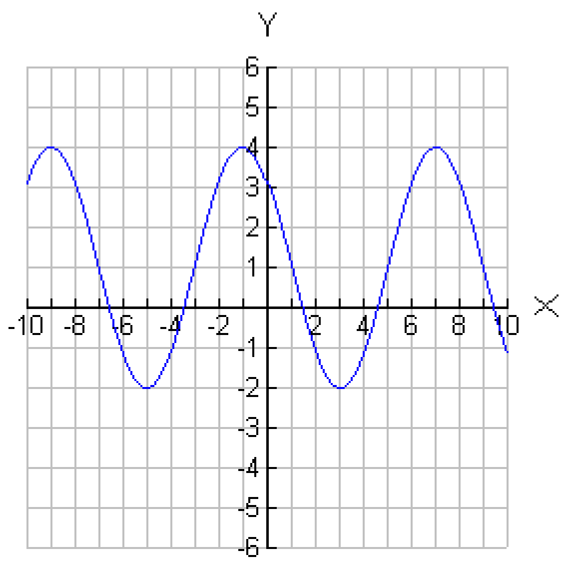
$\csc\theta$

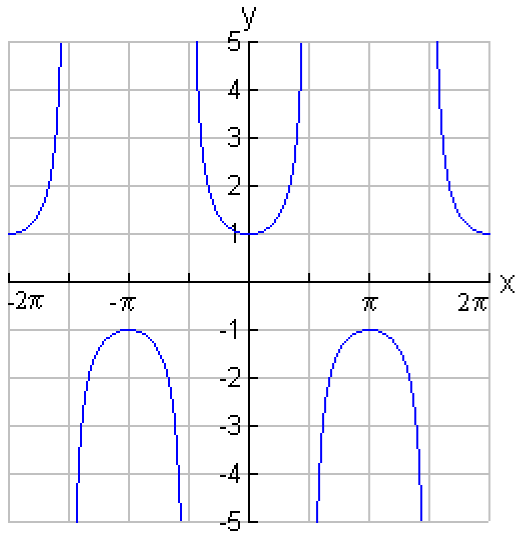
$\cot\theta$

A2.CN.1c

Write an equation for a trigonometric function that matches each of the following graphs. Check your answers with a partner. If different equations have been obtained, confirm by graph or table, the accuracy of each equation.







A2.CN.2 **Understand the corresponding procedures for similar problems or mathematical concepts**

A2.CN.2a
Solve the following inequalities graphically. Express the solutions correct to the nearest hundredth. Explain how the solutions to the two problems are related.

$$x^2 - 3x \geq 8 + y$$
$$x^2 - 3x < 8 + y$$

Students will understand how mathematical ideas interconnect and build on one another to produce a coherent whole.

A2.CN.3 **Model situations mathematically, using representations to draw conclusions and formulate new situations**

A2.CN.3a
Stretch your arms out as wide as you can. Have someone measure across your back the length (in inches) of your arm span, from the tip of your fingers on one hand to the tip of your fingers on your other hand. Have the person also measure your height in inches. Share this information with your classmates and aggregate all of the data.

Enter the information in two lists on your calculator. In the first list, enter the heights, in inches, of all of the participants. In the second list, enter the arm span, in inches, of all of the participants. Make a scatter plot, where the independent variable is height, in inches, and the dependent variable is arm span, in inches. What type of regression function would best model this data? Why?

Write the regression equation with all constants rounded to the nearest tenth.

Using this equation, determine the arm span of a person who is 5 feet tall.

Research to see what other measurements would form similar relationships.

A2.CN.4 **Understand how concepts, procedures, and mathematical results in one area of mathematics can be used to solve problems in other areas of mathematics**

A2.CN.4a
Anthony's teacher told the class that a unit circle has a circumference of 2π. This confused him because he thought a circle has $360°$. Since Anthony is your friend, you would like to help him understand what the teacher means. Write a detailed explanation that compares degrees to radians. Be as thorough as possible in order to help Anthony understand the connection. Include anything that might make this clear, such as diagrams, equations, and so forth.

A2.CN.5 Understand how quantitative models connect to various physical models and representations

A2.CN.5a

The angle of inclination of the sun changes throughout the year. This changing angle affects the heating and cooling of buildings. The overhang of the roof of a house is designed to shade the windows for cooling in the summer and allow the sun's rays to enter the house for heating in the winter.

The sun's angle of inclination at noon can be modeled by the formula:

$$\text{Angle of inclination (in degrees)} = -23.5\cos\left(\frac{360}{365}(x+10)\right)+47$$

for Albany, NY, where *x* is the number of days elapsed in the day of the year, with January first represented by $x=1$, January second represented by $x=2$, and so on.

Find the sun's angle of inclination at noon on Valentine's Day.

Sketch a graph illustrating the changes in the sun's angle of inclination throughout the year.

On what date of the year is the angle of inclination at noon the greatest in Albany, NY?

A2.CN.5b
Mrs. Frost has 1000 bushels of corn to sell. Presently the market price for corn is $\$5.00$ a bushel. She expects the market price to increase by $\$0.15$ per week. For each week she waits to sell, she loses ten bushels due to spoilage.

Determine a function to express Mrs. Frost's total income from selling the corn. What is a reasonable domain and range for this function?

Graph the function and determine when Mrs. Frost should sell the corn to maximize her income. Show your answer is correct by computing the income one week earlier and one week later and showing these values would result in less income.

Students will recognize and apply mathematics in contexts outside of mathematics.

A2.CN.6 Recognize and apply mathematics to situations in the outside world

A2.CN.6a
With a partner, research to find three situations that can be modeled by exponential equations. Write a description of each situation, write and solve one problem based on each of the three situations, and prepare to present the solution to the problem.

A2.CN.6b
You just decided to save money each week in order to buy an expensive cell phone. You decide to start by saving $15 from your paycheck and then save $22 the second week and $29 the third week. You continue this pattern until you have saved enough money.

Use sigma notation to write the indicated sum of this series.

If the cell phone you want to purchase costs $585, how many weeks will you need to save enough money?

Show or explain fully how you arrived at your answer.

A.2.CN.6c
Madison was determined to help clean her local park. She collected one bag of trash the first week, 2 bags the second week, 3 bags the third week, and so on.
Assuming she continues this process, how many bags of trash will she collect in 26 weeks?

How many bags of trash will she collect in *n* weeks?

A2.CN.7 **Recognize and apply mathematical ideas to problem situations that develop outside of mathematics**

A2.CN.7a
Use two thermometers, one Celsius and the other Fahrenheit, to measure the temperature of your hand. Place both thermometers in the palm of your hand and gently squeeze your hand shut. (If you have a temperature probe for your calculator or computer, it may be used instead.)

Share this information with your classmates, and aggregate all of the data.
Enter the information in two lists on your calculator. In the first list, enter the Celsius temperatures. In the second list, enter the Fahrenheit temperatures.
Make a scatter plot, where the independent variable is degrees Celsius, and the dependent variable is degrees Fahrenheit.
Find the linear regression equation that best fits the data.
What is the correlation coefficient? Why would you expect its value to be close to one?

A2.CN.7b
Determine whether each of the following situations would require calculating a permutation or a combination. Explain your answer.

Selecting a lead and an understudy for a school play
Assigning students to their seats on the first day of school
Selecting three students to represent the school at an Honor Society conference in Washington, D.C.

2A.CN.8 **Develop an appreciation for the historical development of mathematics**

A2.CN.8a
Research to find out how calculations with logarithms were done before the invention of calculators.

Back to top

Representation Strand

Students will create and use representations to organize, record, and communicate mathematical ideas.

A2R.1 **Use physical objects, diagrams, charts, tables, graphs, symbols, equations, or objects created using technology as representations of mathematical concepts**

A2.R.1a
Take a cup of very hot water, and measure the temperature of the water. Place the cup in a safe place, and measure the temperature every five seconds for a total of one minute. Record this information in your notebook.

Enter the information in two lists on your calculator. In the first list, enter the time, in seconds, since the experiment began. In the second list, enter the temperature of the water for each indicated time.
Make a scatter plot, where the independent variable is time, in seconds, and the dependent variable is temperature. What type of regression function would best model this data? Why?
Write the regression equation with all constants rounded to the nearest thousandth.
Using this equation, determine the temperature of the water after 25 seconds has passed.
Determine the temperature of the water after two hours has passed. Do both of these answers seem logical to you? Why?

(N.B. If you have a temperature probe for your calculator or computer, it may be used to conduct this experiment.)

A2.R.2 Recognize, compare, and use an array of representational forms

A2.R.2a
Explain clearly in words and with a pair of graphs, the differences between the function $y = \sin x$ and each of the following functions.

$$y = 2\sin x$$
$$y = \sin(2x)$$
$$y = \sin\left(x + \frac{\pi}{2}\right)$$
$$y = \sin x + 3$$

After examining the four sets of graphs, graph one sine function containing all four transformations. Write the equation of the new graph.

A2.R.2b
Give the definition of a circle.
Consider the general equation of a circle, $x^2 + y^2 + Cx + Dy + E = 0$
State the circle in the form $x^2 + y^2 + Cx + Dy + E = 0$ with center $(4, -3)$ and radius 7.
Start with the circle, $x^2 + y^2 - 8x + 4y + 16 = 0.$ Find its center and radius and graph the circle.
Start with the circle $x^2 + y^2 + 10x - 6y + 30 = 0$. Find its center and radius and graph the circle.
How do the coordinates of the center of a circle relate to *C* and *D* when the equation of the circle is in the form $x^2 + y^2 + Cx + Dy + E = 0$?

A2.R.3 Use representation as a tool for exploring and understanding mathematical ideas

A2.R.3a
Use the technique of completing the square to write the equation $x^2 - 6x + 11 = y$ in an appropriate form to find the transformations that produce the graph of the given equation from the graph of the equation $y = x^2$.

Students will select, apply, and translate among mathematical representations to solve problems.

A2.R.4 Select appropriate representations to solve problem situations

A2.R.4a
Solve the following problems using direct or inverse variation as appropriate:

Carlos drove from his home to Union College in 4 hours at 55 mph. How long would it take him if he had traveled at 65 mph?

Keisha earned $44.70 for working 6 hours. How much does she earn for working 8.5 hours?

A2.R.5 Investigate relationships between different representations and their impact on a given problem

A2.R.5a
A company offers its employees a choice of two salary schemes (A and B) over a period of ten years. Scheme A offers a starting salary of $33,000 in the first year and an annual increase of $1200 per year. Scheme B offers a starting salary of $30,000 in the first year and an annual increase of 7% of the previous year's salary. An employee about to join this company hires you to examine the two schemes. Decide which scheme is better financially for the employee. Prepare a variety of ways to present the information to the client, such as tables and graphs.

Students will use representations to model and interpret physical, social, and mathematical phenomena.

A2.R.6 Use mathematics to show and understand physical phenomena (e.g., find the height of a building if a ladder of a given length forms a given angle of elevation with the ground)

A2.R.6a
The table shows the amount of medicine for treating a disease in the bloodstream over the 9 hours following a dose of 10 mg. It seems that the rate of decrease of medicine is approximately proportional to the amount remaining.

Time(hours)	0	1	2	3	4	5	6	7	8
Drug Amount (mg)	10	8.3	7.2	6.0	5.0	4.4	3.7	2.8	2.5

Use this information to find a suitable function to model this data.

Using your model, when will there be less than 1 mg. of medicine in the patient's blood stream?
If the initial dose was 15 mg, when would the amount of medicine in the bloodstream, fall below 5 mg?

A2.R.6b
According to the National Weather Service, the 2005 average monthly temperature, in degrees Fahrenheit, at Central Park in New York City is given below:

Month	Jan.	Feb.	March	April	May	June	July	Aug.	Sept.	Oct.	Nov.	Dec.
Average Temperature (°F)	31.3	36.5	39.4	55.1	58.9	74.0	77.5	79.7	73.3	57.9	49.6	35.3

Write a sinusoidal function that models the average monthly temperature, using $t = 1$ to represent January. According to your model, what is the average temperature in December? Explain the discrepancy from your model to the average monthly temperature in December. How could the model be improved?

A2.R.7 Use mathematics to show and understand social phenomena (e.g., interpret the results of an opinion poll)

A2.R.7a
Search in newspapers and magazines and then write a brief but thorough description of a study that could be characterized as:

a survey
an observational study
a designed experiment

A2.R.8 Use mathematics to show and understand mathematical phenomena (e.g., use random number generator to simulate a coin toss)

A2.R.8a
Use a random number generator to generate thirty random numbers that will simulate tossing a fair coin thirty times. Let an even number represent the result of the coin coming up heads, and an odd number represent the result of tails. Generate a second list of thirty random numbers to represent the second toss of a fair coin. Use the two lists of random numbers to compute the following empirical probabilities: P(head, tail), P(both the same), P(both different). Compute the following theoretical probabilities for the same experiment: P(head, tail), P(both the same), P(both different). How do the empirical and theoretical probabilities compare?

Back to top

Number Sense and Operations

Students will understand meanings of operations and procedures, and how they relate to one another.

Operations

A2.N.1 Evaluate numerical expressions with negative and/or fractional exponents, without the aid of a calculator (when the answers are rational numbers)

A2.N.1a

Find the value of $x^0 + x^{-\frac{1}{2}}$ when $x = 25$.

A2.N.1b

Evaluate $\left(x^{-1} + x^0\right)^{-\frac{3}{2}}$ when $x = \frac{1}{3}$.

A2.N.1c

If $f(x) = 4x^{-\frac{1}{2}}$, find $f(9)$.

A2.N.2 Perform arithmetic operations (addition, subtraction, multiplication, division) with expressions containing irrational numbers in radical form

A2.N.2a

Find the sum of $\sqrt{3}$ and $\sqrt{12}$ in simplest form.

A2.N.2b

Express in terms of $\sqrt{2}$:

$$2\sqrt{50} - \sqrt{32}.$$

A2.N.3 Perform arithmetic operations with polynomial expressions containing rational coefficients

A2.N.3a

Find the difference:

$$\left(\frac{3}{2}x^2 - \frac{2}{3}x + 2\right) - \left(\frac{1}{2}x^2 + \frac{1}{3}x - 5\right)$$

A2.N.3b

Find the product:

$$\left(\frac{1}{3}x - \frac{1}{2}\right)\left(\frac{1}{3}x + \frac{1}{2}\right)$$

A2.N.4 Perform arithmetic operations on irrational expressions

A2.N.4a

Expand the expression $(\pi + \sqrt{5})^2$

A2.N.4b

Express as a single term of $\sqrt{2}$:

$$\frac{4}{\sqrt{8}} + \sqrt{18}$$

A2.N.5 **Rationalize a denominator containing a radical expression**

A2.N.5a

Given $\frac{3+\sqrt{2}}{5-\sqrt{2}}$, write an equivalent expression with a rational denominator.

A2.N.6 **Write square roots of negative numbers in terms of *i***

A2.N.6a

Simplify the problems below. Your friend was absent. Write your friend a complete description of the steps necessary to simplify the problems.

$\sqrt{-7}$

$\sqrt{-80}$

$\sqrt{-2a^2b^5}$

$-6\sqrt{-32x^3}$

A2.N.7 **Simplify powers of *i***

A2.N.7a

Simplify each of the following:

i^1	i^5
i^2	i^6
i^3	i^7
i^4	i^8

Can you see a pattern? If so, what conjecture can you make about the powers of i?

Based on your conjecture, simplify the following:

i^{14} $\quad$ i^{27} $\quad$ i^{301}

A2.N.8 **Determine the conjugate of a complex number**

A2.N.8a

Sally's math teacher said you could use the conjugate of a complex number to rationalize the denominator of a fraction which contains a complex number. Sally is trying to rationalize the denominator of $\frac{4+i}{3+2i}$ but cannot remember if $-3-2i$ or $3-2i$ is the conjugate. Which one is the conjugate and why?

A2.N.8b

Research and define: complex number, imaginary number, real number, imaginary part, real part, conjugate

When speaking of a complex number what do *a, b,* and *i* represent or mean?

Give an example and draw a graphical representation of a complex number that fits each description:

$a \neq 0$ and $b \neq 0$

$a = 0$ and $b \neq 0$

$a \neq 0$ and $b = 0$

Write the conjugate of each of the complex numbers, and draw the graphical representation of each conjugate.

How are the graphical representations of real numbers and complex numbers the same and how are they different?

How are the graphical representations of a complex number and its conjugate the same and different?

A2.N.9 **Perform arithmetic operations on complex numbers and write the answer in the form $a+bi$** ***Note: This includes simplifying expressions with complex denominators.***

A2.N.9aSimplify the following:

$(2+3i)+(-1+6i)$

$(7-i)-(3i+4)$

$(4+5i)^2$

$\frac{2+9i}{1-i}$

A2.N.10 **Know and apply sigma notation**

A2.N.10a

Find the value of $\sum_{i=1}^{3} i^2 - i$

A2.N.10b
Adrianna decided to save money each week in order to buy an expensive sound system. She decide to start by saving $15 from her first paycheck, and then save $22 the second week and $29 the third week. She continued this pattern until she had saved enough money.

Use sigma notation to write the indicated sum of this series.
If the sound system she wanted to purchase costs $585, how many weeks will she need to save enough money?
Explain how you arrived at your answer.

Back to top

Algebra

Students will represent and analyze algebraically a wide variety of problem solving situations.

Equations and Inequalities

A2.A.1 **Solve absolute value equations and inequalities involving linear expressions in one variable**

A2.A.1a
Solve and check:

$|2m-4|=8$

$\left|\frac{x}{2}+6\right|\le 10$

$-3|2t|>18$

$|w+4|=-6$

A2.A.1b
Last year's senior class spent $23.95 for each yearbook. This year's yearbook committee knows that their yearbook must be within $5.50 of last year's favor. Write an absolute value inequality that could be used to model the acceptable price range for this year's yearbook, and then solve the inequality to find the range of acceptable prices for the yearbook.

A2.A.2 **Use the discriminant to determine the nature of the roots of a quadratic equation**

A2.A.2a
Describe the nature of the roots of the quadratic equation whose discriminant is:

144
– 36
27
0

A2.A.2b
For each of the following values of the discriminant, state the number of *x*-intercepts the graph would have, and sketch a graph of the parabola:

144
– 6
27
0

A2.A.2c
For each of the following:

Sketch a graph of the function.
Set the function equal to 0 and solve.
Find the discriminant.

What connections can you make between the discriminant, the solution(s), and the graph of the function?

$y = x^2 - x - 6$

$y = x^2 - 2x - 15$

$y = x^2 - 2$

$y = 3x^2 - 5x - 6$

$y = x^2 - 8x + 16$

$y = x^2 + 2x + 1$

A2.A.3 **Solve systems of equations involving one linear equation and one quadratic equation algebraically** ***Note: This includes rational equations that result in linear equations with extraneous roots.***

A2.A.3a
Solve the following systems of equations algebraically and check all solutions:

1.) $x^2 - 14 = y$
$y + 1 = 2x$

2.) $x^2 + y = 8$
$y = x$

3.) $\dfrac{y}{x^2 - 9} = 1$
$y - 3 = x$

A2.A.4 Solve quadratic inequalities in one and two variables, algebraically and graphically

A2.A.4a
Solve the following inequalities algebraically, and then check the solutions graphically:

1.) $x^2 - 25 \geq 0$
2.) $5y^2 < 10y$
3.) $2m^2 \leq m + 21$

A2.A.4b
Solve the following inequalities graphically. Express the solutions rounded to the nearest hundredth. Explain how the solutions to the two problems are related.

$x^2 - 3x \geq 8 + y$

$x^2 - 3x < 8 + y$

A2.A.5 Use direct and inverse variation to solve for unknown values

A2.A.5a
Use direct or inverse variation to solve for the unknown values:

If *p* varies directly as *q*, and *p* = 7 when *q* = 9, find *p* when *q* = 12.
If *m* varies inversely as *t*, and *m* = 5 when *t* = 6, find *t* when *m* = 10.

A2.A.5b
Solve the following problems using direct or inverse variation as appropriate:

1.) Carlos drove from his home to Union College in 4 hours at 55 mph. How long would it have taken him if he had traveled at 65 mph?
2.) Keisha earned $44.70 for working 6 hours. How much will she earn for working 8 hours?

A2.A.6 Solve an application which results in an exponential function

A2.A.6a
Brianna decided to invest her $500 tax refund rather than spending it. She found a bank that would pay her 4% interest, compounded quarterly. If she deposits the entire $500 and does not deposit or withdraw any other amount, how long will it take her to double her money in the account?

Students will perform algebraic procedures accurately.

Variables and Expressions

A2.A.7 Factor polynomial expressions completely, using any combination of the following techniques: common factor extraction, difference of two perfect squares, quadratic trinomials

A2.A.7a
Factor each of the expressions completely. Discuss the process that you followed to obtain the answers.

$4s^4 - 64$

$a^8 - b^4$

$3x^2 - 6x - 24$

$5y^3 + 15y^2 - 20y$

A2.A.8 Apply the rules of exponents to simplify expressions involving negative and/or fractional exponents

A2.A.8a
Simplify:

$$(3a^{-4}b^{\frac{5}{3}})(a^{-2}b^{\frac{1}{3}})$$

A2.A.8b
Simplify:

$$\frac{x^{\frac{1}{2}}}{x^{-\frac{1}{2}}}$$

A2.A.9 Rewrite algebraic expressions that contain negative exponents using only positive exponents

A2.A.9a
Rewrite each of the following expressions, using only positive exponents, with none of the variables equal to zero:

$$\left(3xy^{-3}\right)^{-2}$$

$$\frac{a^{-2}b^{3}c^{0}}{a^{-3}b^{5}c}$$

$$\frac{3f^{-2}g^{-3}}{2^{-1}f^{4}g^{-4}}$$

A2.A.10 Rewrite algebraic expressions with fractional exponents as radical expressions

A2.A.10a
Rewrite the three expressions below, using radicals. All of the variables are positive numbers.

$$2j^{\frac{1}{3}}$$

$$\left(5v\right)^{\frac{3}{4}}$$

$$\frac{5}{u^{-\frac{4}{5}}}$$

A2.A.11 Rewrite algebraic expressions in radical form as expressions with fractional exponents

A2.A.11a
Rewrite the radical expressions below, using exponents. All of the variables are positive numbers.

$$\sqrt{xy^{3}}$$

$$\sqrt[3]{8a^{4}b^{5}}$$

$$\frac{5}{\sqrt[4]{r^{2}s^{3}}}$$

A2.A.12 Evaluate exponential expressions, including those with base e

A2.A.12a

Find the exact value of $81^{-\frac{3}{4}}$.

A2.A.12b

Evaluate $e^{2\ln 3}$

A2.A.12c

Find the value of $9x^{-\frac{1}{2}}$ if $x = 4$.

A2.A.13 Simplify radical expressions

A2.A.13a

Simplify:

$$\sqrt{72x^4y^3}.$$

A2.A.13b

Simplify:

$$\sqrt[3]{24x^5y^2}\ .$$

A2.A.14 Perform addition, subtraction, multiplication, and division of radical expressions

A2.A.14a

Find the sum of $\sqrt{12}$ and $\sqrt{75}$ in simplest form.

A2.A.14b

Express in terms of $\sqrt{2}$:

$$2\sqrt{18}\ -\ \sqrt{8}\ .$$

A2.A.14c

Perform the indicated operations, and express your answer in simplest form:

$$\sqrt[3]{24x^2}\cdot\sqrt[3]{16x^7}$$

$$\frac{12\sqrt{20w}-36\sqrt{35w}}{-6\sqrt{5w}}$$

A2.A.15 Rationalize denominators involving algebraic radical expressions

A2.A.15a

Express as a fraction with a rational denominator:

$$\frac{7}{4-\sqrt{2}}$$

A2.A.15b

Express as a single term of $\sqrt{3}$:

$$\frac{4}{\sqrt{12}}+\sqrt{48}$$

A2.A.16 Perform arithmetic operations with rational expressions and rename to lowest terms

A2.A.16a
Perform the indicated operation and express in lowest terms:

$$\frac{3}{a+2}-\frac{a}{2-a}+\frac{2}{a^2-4}$$

A2.A.16b
Perform the indicated operation and express in lowest terms:

$$\left(\frac{a+b}{a}\right)\left(\frac{ab}{a^2+3ab+2b^2}\right)$$

A2.A.16c
Perform the indicated operation and express in lowest terms:

$$\frac{x^2+3x}{x-2}\div\frac{x^2+4x+3}{x^2-x-2}$$

A2.A.17 Simplify complex fractional expressions

A2.A.17a
Simplify:

$$\frac{\dfrac{x^2-25}{x^2-16}}{\dfrac{2x+10}{x^2-4x}}$$

A2.A.18 Evaluate logarithmic expressions in any base

A2.A.18a
Evaluate each of the following without using a calculator:

$$\frac{\log 100}{\log 10}$$

$$\frac{\log_3 9}{\log_2 8}$$

$$\log_4 64\cdot\log_2 16\cdot 3\log_3 81$$

A2.A.18b
Evaluate each of the following to the nearest hundredth:

$$\frac{2\log 16}{\log 4+\log 14}$$

$$\frac{\ln 11}{\ln 32-\ln 27}$$

$$\frac{\log_3 81+\log_4 256}{\log_2 64}$$

A2.A.19 Apply the properties of logarithms to rewrite logarithmic expressions in equivalent forms

A2.A.19a
Express as a single logarithm:

$$\log_b a + 4\log_b c$$
$$\log_a 8 - \log_a 4 - 2\log_a 3$$

A2.A.19b
Expand each of the following:

$$\log_n x^2 y^3 z^4$$

$$\log_3 \sqrt{5x^3}$$

$$\ln 3e^{2x}$$

A2.A.19c
Lauren and Diana disagree about one of the rules for simplifying logarithms.

Lauren says that $\log A + \log B = \log(A+B)$ because you can factor out the log.

Diana says that $\log A + \log B = \log(AB)$, because you add exponents when you are multiplying.

Which student is correct?

Explain your answer using two different strategies (e.g., use a table, a graph, an algebraic proof, etc.).

Equations and Inequalities

A2.A.20 Determine the sum and product of the roots of a quadratic equation by examining its coefficients

A2.A.20a
Find the sum of the roots and the product of the roots for the following quadratic equations:

$$x^2 - x - 12 = 0$$
$$y^2 - 2y = 3y + 50$$
$$3z^2 + 2 = 7z$$
$$r^2 + 4r + 4 = 2r$$

A2.A.20b
Given the quadratic equation $w^2 + 5w + k = 0$:

What is the sum of the roots?
What is the product of the roots?
If one of the roots of the equation is -3, what is the other root?

A2.A.21 Determine the quadratic equation, given the sum and product of its roots

A2.A.21a
Write a quadratic equation whose roots have a sum of -4 and a product of 4.

A2.A.21b
Write a quadratic equation whose roots have a sum of 4 and a product of -4, and find the roots of this equation.

A2.A.21c
If a quadratic equation has the roots given below, find the sum of the roots, find the product of the roots, and write a quadratic equation with the given roots.

3 and -6

$\frac{1}{2}$ and $\frac{2}{3}$

$1+\sqrt{2}$ and $1-\sqrt{2}$

$2+3i$ and $2-3i$

A2.A.21d
Solve each quadratic equation and use the roots to complete the table:

Quadratic Equation	Roots	Sum of Roots	Product of Roots
$x^2-9=0$			
$x^2-2x-8=0$			
$x^2-16x+64=0$			
$2x^2+5x+3=0$			
$6x^2-7x-5=0$			
$x^2-9x=0$			

Use the information in the table to make a conjecture about the relationship between a quadratic equation and the sum and product of its roots. Use correct mathematical language to write your conjecture.

When finished writing, exchange the paper with another student. Read the conjecture and decide whether the conjecture is valid.
If it is valid, write questions that will help the student prove that the conjecture is valid.
If the conjecture is invalid, write questions that challenge the conjecture based upon correct mathematics and mathematical reasoning.

Return the paper. Respond to the student's questions using correct mathematical language and reasoning to prove or disprove the validity of the conjecture.

A2.A.22 Solve radical equations

A2.A.22a
Solve and check:

$$n=1+\sqrt{21-n}\,.$$

A2.A.22b
Is {1, -2} the solution set of the equation $x=3+\sqrt{11-7x}$? Explain your answer.

A2.A.22c
Determine the number of solutions for each of the following two equations:

$$\sqrt{8y-2}=\sqrt{5y+10}$$

$$2+\sqrt{3r+10}=1$$

Explain how to determine the number of solutions to a radical equation without actually solving an equation.

A2.A.22d
The sum of a number and its square root is equal to six. Find the number.

A2.A.23 Solve rational equations and inequalities

A2.A.23a

Solve and check: $\frac{7}{t-5}+\frac{2}{3}=\frac{t+5}{t-5}$.

A2.A.23b

Solve and check: $\frac{r}{r+3}+\frac{4}{r-2}=\frac{20}{r^2+r-6}$.

A2.A.23c

Solve the inequality: $\frac{3v+7}{v+3}>2$.

A2.A.23d

Dante has to travel from Cambridge, New York to Buffalo, New York, a distance of approximately 333 miles. He estimates that he can average 20 mph faster during the 273 miles that he will be driving on a main highway than he can when he drives on back roads. If he wants to complete the trip in eight hours, find, to the nearest integer, the rate that Dante must travel on the main highway.

A2.A.24 Know and apply the technique of completing the square

A2.A.24a

Solve the following equations by completing the square:

$$x^2+4x+3=0$$

$$2t^2-7t=4$$

$$3m^2-12m-60=3$$

A2.A.24b

One of the students in class was absent the day the class learned the technique of completing the square. Write an explanation of how to solve the following equation using the technique of completing the square that you could give to the student who had been absent:

$$x^2+8x-3=0$$

A2.A.24c

Use the technique of completing the square to write the equation $x^2-6x+11=y$ in an appropriate form to find the transformations that produce the graph of the given equation from the graph of the equation $y=x^2$.

A2.A.25 Solve quadratic equations, using the quadratic formula

A2.A.25

Solve each of the following equations using the quadratic formula:

$$2x^2-5x=12$$

$$r^2-7r+3=0$$

$$y-3=2y^2$$

$$1+\frac{3}{x^2}=\frac{5}{x}$$

A2.A.26 **Find the solution to polynomial equations of higher degree that can be solved using factoring and/or the quadratic formula**

A2.A.26a
Solve the following equations. Express any irrational solutions in simplest radical form.

$$x^4 = 13x^2 - 36$$
$$t^5 - 10t^3 + 21t = 0$$
$$\left(x^2 + 5x - 7\right)\left(x + 2\right) = 0$$

A2.A.27 **Solve exponential equations with and without common bases**

A2.A.27a
Solve the following equations:

$$3^{(2y-4)} = 9^{(3y+2)}$$
$$\left(\frac{1}{2}\right)^{(x-3)} - 32 = 0$$
$$4^{2t} = 8^{(t+4)}$$

A2.A.27b
Solve the following equations. Express solutions correct to the nearest hundredth.

$$5^x = 3$$
$$2^{(y+1)} - 3 = 8.72$$

A2.A.28 **Solve a logarithmic equation by rewriting as an exponential equation**

A2.A.28a
Solve the following equations:

$$\log x = -2$$
$$\log_4 x = 1\frac{1}{2}$$
$$\log_x 36 = 2$$
$$\log_6 x + \log_6 (x-2) = 1$$

Students will recognize, use, and represent algebraically patterns, relations, and functions.

Patterns, Relations, and Functions

A2.A.29 **Identify an arithmetic or geometric sequence and find the formula for its *n*th term**

A2.A.29a
Maya has decided to train for a marathon (26 miles) and has set up a practice schedule to build her stamina. When she began she was able to run 3 miles, but she intends to train every day and increase her run by 2 miles each week. Find a pattern and write a formula that will give the number of miles Maya can run in week *n*.

Using the formula, how many weeks will Maya need to train in order to be ready for the marathon?

A2.A.29b
Dahar is taking a class in word processing and is trying to increase his typing speed. When he began, he could type 20 words per minute. He practiced faithfully every day and noticed an increase in his speed of 10% per week. Find a pattern and write a formula that will give the number of words per minute Dahar can type in *n* weeks.

Using the formula, how many weeks will Dahar need to practice in order to type 60 words per minute?

A2.A.30 Determine the common difference in an arithmetic sequence

A2.A.30a
What is the common difference in the following arithmetic sequence?

5, 9, 13, 17,…

$$0.5,\ 3.5,\ 24.5,\ 171.5,\ldots$$

A2.A.30b
What is the common difference in the arithmetic sequence defined by the following formula?

$$t_n = 6n + 3$$

A2.A.31 Determine the common ratio in a geometric sequence

A2.A.31a
What is the common ratio in the following geometric sequence?

$$6,\ -3,\ \frac{3}{2},\ -\frac{3}{4},\ldots$$

A2.A.31b
What is the common ratio in the geometric sequence defined by the following formula?

$$t_n = 5(3)^{n-1}$$

A2.A.32 Determine a specified term of an arithmetic or geometric sequence

A2.A.32a
Find the specified term of each of the following arithmetic sequences.

t_{20}:

$$1,\ \frac{1}{2},\ 0,\ -\frac{1}{2},\ldots$$

a_7:

$a_1 = 3x$

$d = -x$

t_{25}:

$t_3 = -4 + 5i$

$t_6 = -13 + 11i$

A2.A.32b
Find the specified term of each of the following geometric sequences.

a_{18} :

$0.5,\ 3.5,\ 24.5,\ 171.5, \ldots$

t_{31} :

$t_1 = 14$

$r = 3.1$

A2.A.33 Specify terms of a sequence, given its recursive definition

A2.A.33a
Use the recursive rule given to write the first four terms of each sequence.

$$a_1 = -2$$
$$a_{n+1} = (a_n)^2 + 3$$

$$t_1 = 3x$$
$$t_n = \frac{t_{n-1} + 2}{n-1}$$

A2.A.34 Represent the sum of a series, using sigma notation

A2.A.34a
Use sigma notation to represent the sum of the following series.

$3 + 6 + 9 + 12 + \ldots$ for the first 33 terms.

$-3 + 6 - 12 + 24 \ldots$ for the first 50 terms.

$6 + 2 + \frac{1}{3} + \frac{1}{6} \ldots$ for *n* terms.

A2.A.35 Determine the sum of the first *n* terms of an arithmetic or geometric series

A.2.A.35a
Madison was determined to help clean her local park. She collected one bag of trash the first week, 2 bags the second week, 3 bags the third week, and continued at the same rate.

Assuming she continues this process, how many bags of trash will she collect in 26 weeks?

How many bags of trash will she collect in *n* weeks?

If Madison collected one bag of trash the first week, 2 bags the second week, 4 bags the third week, and continued at the same rate, how many bags of trash would she collect in 26 weeks?

A2.A.35b
Find the indicated sum of the series.

$$\sum_{k=1}^{n} 4(0.5)^k$$

A2.A.35c
Given the sequence $-4,\ 0,\ 4,\ 8,\ 12\ldots$, Paul notices a pattern and finds a formula he believes will find the sum of the first *n* terms. His formula is $\sum_{i=1}^{n} 4n-8=2n^2-6n$. Show that Paul's formula is correct.

A2.A.35d
A company offers its employees a choice of two salary schemes (A and B) over a period of ten years. Scheme A offers a starting salary of $33,000 in the first year and an annual increase of $1200 per year. Scheme B offers a starting salary of $30,000 in the first year and an annual increase of 7% of the previous year's salary. An employee about to join this company hires you to examine the two schemes. Decide which scheme is better financially for the employee. Prepare a variety of ways to present the information to the client, such as tables and graphs.

A2.A.36 Apply the binomial theorem to expand a binomial and determine a specific term of a binomial expansion

A2.A.36a
Use the binomial theorem to write the expansion of the following:

$$(3x+2y)^4$$

A2.A.36b
Find the sixth term of the binomial expansion of the following:

$$(a-3b)^9$$

A2.A.37 Define a relation and function

A2.A.37a
Use the definition of a relation to explain why the set {(-3,5), (4,7), (4,5)} is a relation.

A2.A.37b
Use the definition of a function to explain why the set {(-3,5), (4,7), (0,5)} is a function.

A2.A.37c
Which of the following sets are relations? Explain your answer.

{2,4,6, …}
{students}
{(1,2),(3,7)}
$\{x : x+2=5\}$
$\{(x,y) : x+2=y^2\}$

A2.A.37d
Which of the following sets are functions? Explain your answer.

{(-3,7),(-5,9),(0,0),(8,9)}
{(-4,7),(-4,8),(3,4),(11,8)}
$\{(x,y) : x^2+2=y\}$
a list which matches students with their ages

A2.A.38 Determine when a relation is a function

A2.A.38a
Which of the following relations are functions? Explain your answer.

$$y = 7x - 3$$
$$y = x^2 + 3x + 2$$
$$x^2 + y^2 = 9$$
$$y = |x| + 5$$
$$y = \pm\sqrt{x - 3}$$

A2.A.38b
Liza was absent from school and emailed two of her friends to help her understand how to decide if a relation is a function.

Mike said: *Make a table, and see if you get two of the same y-values*.
John said: *Look at the graph. See if a vertical line crosses the graph in more than one place. If it does, then we have a function*.

Which student is correct? Why? Provide a counterexample to explain any errors made by either Mike or John.

A2.A.39 Determine the domain and range of a function from its equation

A2.A.39a
Find the domain and range of each of the following functions:

$$f(x) = -x^2 + 4$$
$$g(x) = 3x - 8$$
$$k(x) = |x + 3|$$
$$m(x) = \sqrt{x - 7}$$
$$h(x) = \frac{x + 2}{x^2 - 9}$$
$$t(x) = \frac{x + 2}{\sqrt{x - 3}}$$

A2.A.39b
Mrs. Frost has 1000 bushels of corn to sell. Presently the market price for corn is $\$5.00$ a bushel. She expects the market price to increase by $\$0.15$ per week. For each week she waits to sell, she loses ten bushels due to spoilage.

Determine a function to express Mrs. Frost's total income from selling the corn. What is a reasonable domain and range for this function?

Graph the function and determine when Mrs. Frost should sell the corn to maximize her income. Show your answer is correct by computing the income one week earlier and one week later and showing these values would result in less income.

A2.A.40 Write functions in functional notation

A2.A.40a
Use function notation to describe each of the following functions:

x	*y*
-3	9
-2	4
-1	1
0	0
1	1
2	4
3	9

$$\{(x, y) : y \text{ is five times } x \text{ increased by } 8\}$$
$$\{(x, y) : y = 9\}$$
$$\{(x, y) : x = y\}$$

A2.A.41 Use functional notation to evaluate functions for given values in the domain

A2.A.41a
Given The Function: $f(x) = 5x - 8$

Evaluate: $f(-1),\ f(6),\ f(0),\ f\left(\frac{2}{3}\right),\ f(a),\ f(-x):$

Given The Function: $g(x) = (x-7)^2$

Evaluate: $f(-1),\ f(6),\ f(0),\ f\left(\frac{2}{3}\right),\ f(a),\ f(-x):$

Given The Function: $h(x) = \dfrac{3}{x-1}$

Evaluate: $f(-1),\ f(6),\ f(0),\ f\left(\frac{2}{3}\right),\ f(a),\ f(-x):$

Given The Function: $t(x) = |3x+4|$

Evaluate: $f(-1),\ f(6),\ f(0),\ f\left(\frac{2}{3}\right),\ f(a),\ f(-x):$

A2.A.42 Find the composition of functions

A2.A.42a
Evaluate each of the following pairs of functions:

$$(f \circ g)(4), (g \circ f)(4), (f \circ g)(x), (g \circ f)(x) \ :$$
$$f(x) = 3x + 1 \quad g(x) = x^2 + 2$$
$$f(x) = 4 \quad g(x) = |x|$$
$$f(x) = 2^x \quad g(x) = x^2 + 3$$
$$f(x) = \frac{x+1}{5} \quad g(x) = 5x - 1$$

A2.A.42b
Is the composition of two functions ever commutative? Explain your answer.

A2.A.42c
John claims that he performs the composition of functions each morning when he puts on his shoes and socks. Is he correct? Explain.

A2.A.43 Determine if a function is one-to-one, onto, or both

A2.A.43a
For each of the following functions, state whether the function is one-to-one, onto, neither, or both:

$f(x) = 3x - 8$

$g(x) = (x+5)^2$

$h(x) = |2x| + 4$

$k(x) = -5$

A2.A.43b
Find a counterexample to refute each of the following claims:

All functions of the form $f(x) = x^n$ are one-to-one.

All one-to-one functions are onto.

A2.A.44 Define the inverse of a function

A2.A.44a
Use the definition of the inverse of a function to explain why $\{(1,4), (2,3)\}$ is the inverse of the function $\{(4,1),(3,2)\}$.

A2.A.44b
Find $(f \circ f^{-1})(4)$.

A2.A.45 Determine the inverse of a function and use composition to justify the result

A2.A.45a
Find the inverse of the following functions. Use composition to show your result is the inverse of the given function.

$f(x) = 3x + 8$

$g(x) = (x+4)^3$

$h(x) = \frac{x+3}{x}$

A2.A.45b
Demonstrate that $f(x) = 2x - 3$ and $g(x) = 0.5x + 1.5$ are inverses using at least two different strategies (numeric, graphic or algebraic).

A2.A.46 **Perform transformations with functions and relations:** $f(x+a)$, $f(x)+a$, $f(-x)$, $-f(x)$, $af(x)$

A2.A.46a

On the same set of axes, graph $y_1 = \sqrt{x}$, $y_2 = \sqrt{x+2}$, and $y_3 = \sqrt{x-3}$. What transformation describes the relationship between y_1 and y_2 ? What transformation describes the relationship between y_1 and y_3 ? Use the answers to these questions to predict what the graph of $y_4 = \sqrt{x+5}$ and $y_5 = \sqrt{x-4}$ will look like, then use a graphing calculator to check your prediction. Write a description of how the graph of $f(x)$ and the graph of $f(x+a)$ are related.

A2.A.46b

Write an equation for the graph of the function, $g(x)$, obtained by shifting the graph of $f(x) = x^2$ three units to the left, stretching the graph vertically by a factor of two, reflecting that result over the *x* – axis, and then translating the graph up four units.

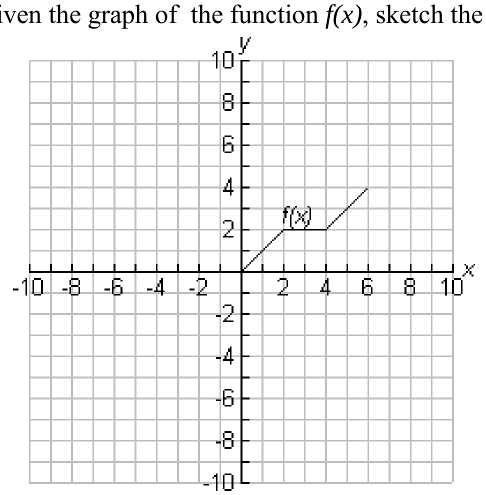
A2.A.46c

Describe the transformations that would produce the graph of the second function from the graph of the first function:

$$f(x) = |x| \qquad g(x) = -3|x| + 8$$
$$f(x) = \sin x \qquad g(x) = 4\sin x$$
$$f(x) = \sqrt{x} \qquad g(x) = \sqrt{-x} - 5$$

A2.A.46d

Given the graph of the function *f(x)*, sketch the graphs of $f(x+1)$, $f(x)-2$, $f(-x)$, $-f(x)$, $2f(x)$:



A2.A.46e

Explain clearly in words and with a pair of graphs, the differences between the function $y = \sin x$ and each of the following functions.

$$y = 2\sin x$$
$$y = \sin(2x)$$
$$y = \sin\left(x + \frac{\pi}{2}\right)$$
$$y = \sin x + 3$$

After examining the four sets of graphs, graph one sine function containing all four transformations. Write the equation of the new graph.

A2.A.47 Determine the center-radius form for the equation of a circle in standard form

A2.A.47a
Use the technique of completing the square to convert the equation $x^2 + y^2 - 4x - 6y + 8 = 0$ into center-radius form. What is the center and what is the radius of this equation?

A2.A.47b
Convert the equation $x^2 + y^2 + 2x - 4y - 11 = 0$ into center-radius form. When is this form of the equation more useful?
Explain how to convert from center-radius form to standard form.

A2.A.47c
Give the definition of a circle.
Consider the general equation of a circle, $x^2 + y^2 + Cx + Dy + E = 0$
State the circle in the form $x^2 + y^2 + Cx + Dy + E = 0$ with center $(4, -3)$ and radius 7.
Start with the circle, $x^2 + y^2 - 8x + 4y + 16 = 0$. Find its center and radius and graph the circle.
Start with the circle $x^2 + y^2 + 10x - 6y + 30 = 0$. Find its center and radius and graph the circle.
How do the coordinates of the center of a circle relate to *C* and *D* when the equation of the circle is in the form $x^2 + y^2 + Cx + Dy + E = 0$?

A2.A.48 Write the equation of a circle, given its center and a point on the circle

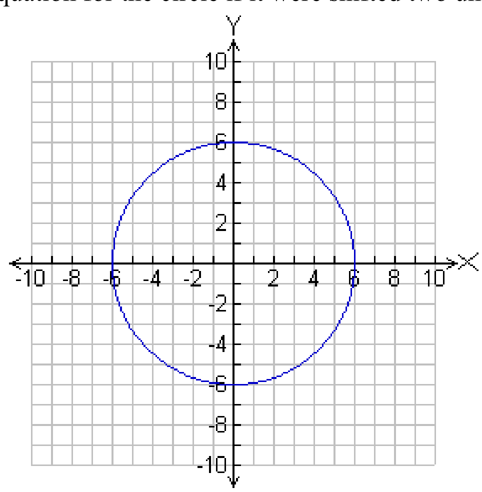
A2.A.48a
Write an equation of a circle with a center that is (3, -2) and passes through the point (-5, 8).

A2.A.48b
Write an equation of a circle whose diameter has endpoints (4, -1) and (-6, 7).

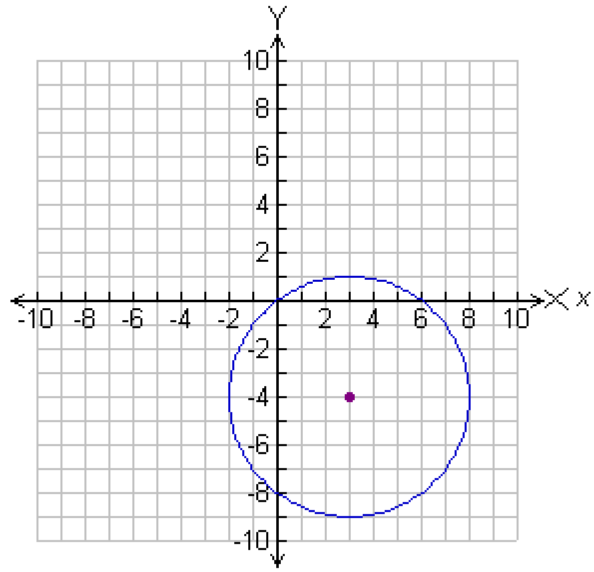
A2.A.48c
Write an equation of a circle that is tangent to the *x*-axis, with a center that is (-2, 4).

A2.A.49 Write the equation of a circle from its graph

A2.A.49a
Write an equation for the circle whose graph is shown below.
Write an equation for the circle if it were shifted two units to the left and three units up.

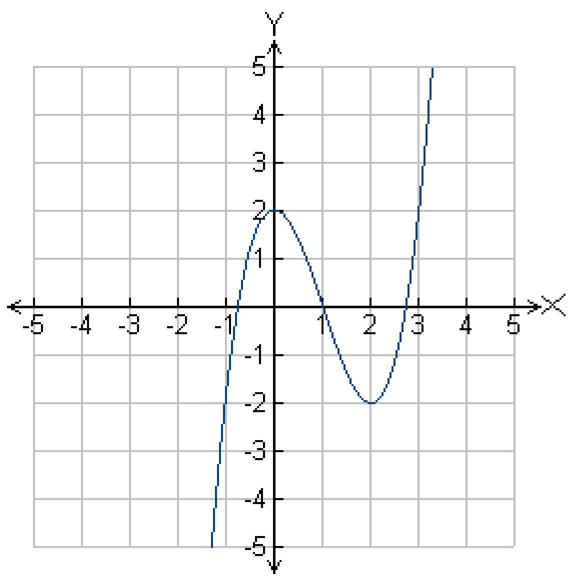


A2.A.49b
Describe the circle whose graph is shown below. Write an equation for the circle and name six points that lie on the circle.



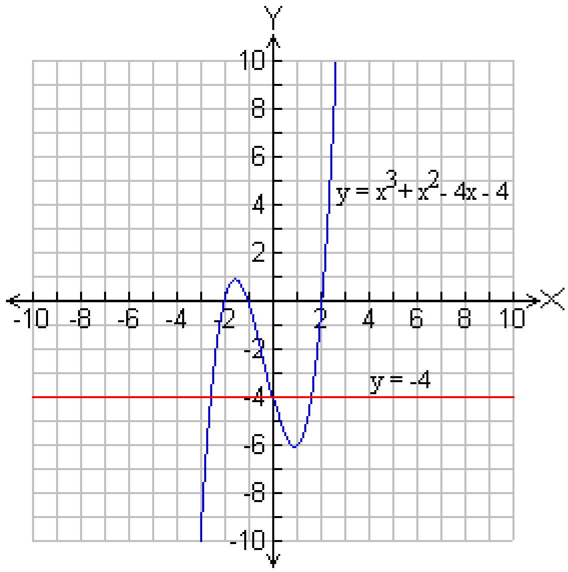
A2.A.50 Approximate the solution to polynomial equations of higher degree by inspecting the graph

A2.A.50a
The function $f(x) = x^3 - 3x^2 + 2$ is graphed below. Use the graph to approximate the solutions to the equation $x^3 - 3x^2 + 2 = 0$



A2.A.50b

Use the graph below to approximate the solutions to the equation $x^3 + x^2 - 4x - 4 = -4$.



A2.A.50c

The Art Club has purchased flat sheets of cardboard to make storage boxes for the club's art supplies. They will make boxes by cutting a square from each corner of the 24 inch by 36 inch sheet of cardboard.

Draw a diagram to illustrate the given information.

Express the volume of the box as a function of the side of the square cut from the cardboard.

Make a table for the function. Use the table to find the volume of a box formed by removing a square which has a 10 inch side, and the length of the side of a square that would produce a volume of 1792 cubic inches.

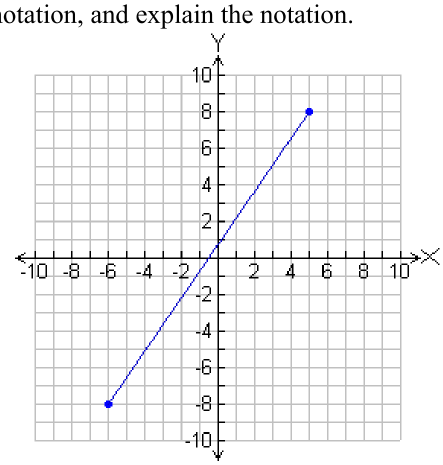
Sketch a graph of the function, and use the graph to find the side of the square that would produce the maximum volume. Find the maximum volume of the box that can be made.

Make a chart containing the side of the square removed, length, width, height and volume of the box created that the club could use for quick reference to make boxes of appropriate size for the supplies. Be sure to determine an appropriate degree of accuracy for the entries in the chart.

A2.A.51 Determine the domain and range of a function from its graph

A2.A.51a

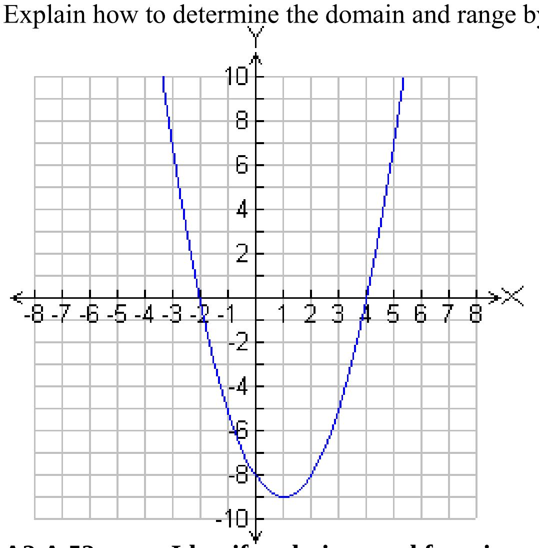
What is the domain and range of the function shown below? Express your answer in standard mathematical notation, and explain the notation.



A2.A.51b

Determine the domain and range of the function $f(x) = x^2 - 2x - 8$ by looking at its graph, shown below.

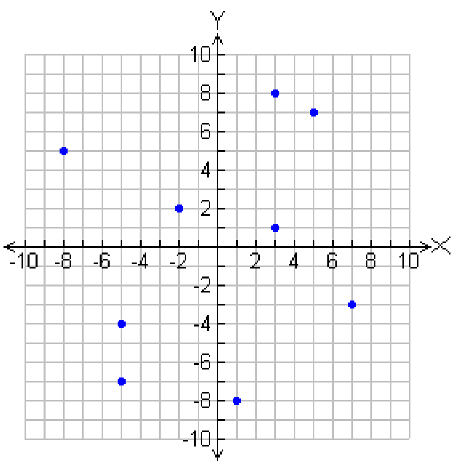
Explain how to determine the domain and range by examining the equation of the function.

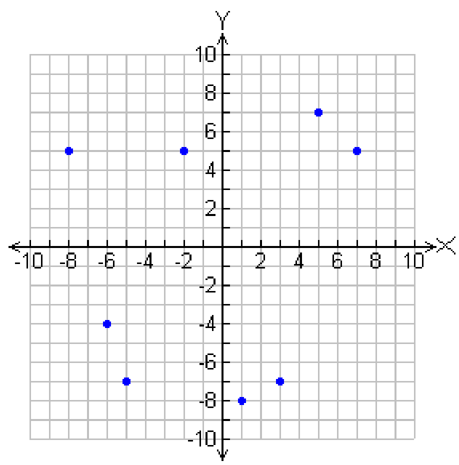


A2.A.52 Identify relations and functions, using graphs

A2.A.52a

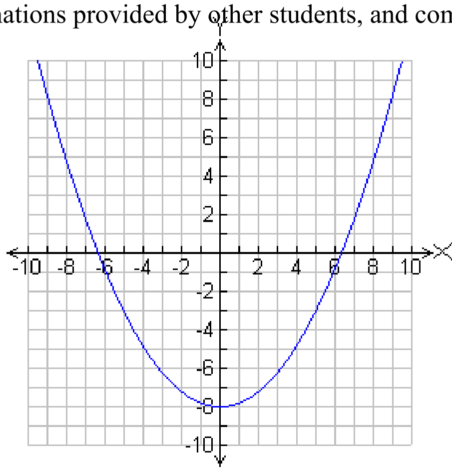
Which of the following two graphs represents a function? Explain your answer.

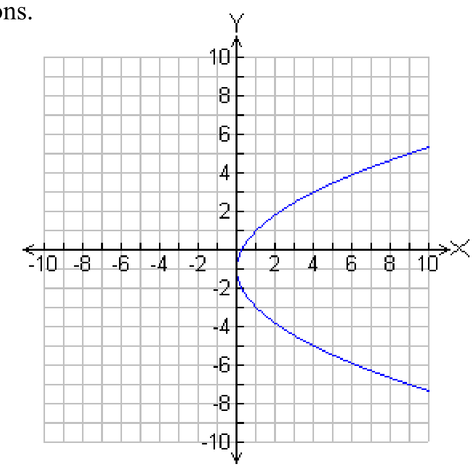




A2.A.52b

Which of the following two graphs represents a function? Explain how you determined your answer. Read explanations provided by other students, and compare solutions.





A2.A.53 **Graph exponential functions of the form $y = b^x$ for positive values of *b*, including $b = e$**

A2.A.53a
Sketch and label the following graphs:

$y = 5^x$.

$y = e^x$

$y = \left(\frac{1}{2}\right)^x$

When b is a positive number, what similarities and differences do the graphs of all forms of $y = b^x$ have?

A2.A.53b
On the same set of axes, use a graphing package or graphics calculator to graph the following functions:
$y = 2^x, y = 10^x, y = 1.3^x, y = e^x$

The functions above are all of the form $y = b^x$.
What effect does changing b values have on the shape of the graph?
What is the *y*-intercept of each graph?
What is the horizontal asymptote of each graph?

A2.A.54 **Graph logarithmic functions, using the inverse of the related exponential function**

A2.A.54a
Sketch the graph of $y = f(x)$ where $f(x) = 2^x$.

On the same axes, sketch the graph of $y = g(x)$, where *g* is the inverse function of *f*.

State the equation for $g(x)$.

Trigonometric Functions
A2.A.55 **Express and apply the six trigonometric functions as ratios of the sides of a right triangle**

A2.A.55a
Sketch right triangle *ABC*, $m\angle C = 90, AC = 9, CB = 12, AB = 15.$ Express the value of each of the following as the ratio of the sides of the triangle.

$\sin A$	$\sin B$
$\cos A$	$\cos B$
$\tan A$	$\tan B$
$\sec A$	$\sec B$
$\csc A$	$\csc B$
$\cot A$	$\cot B$

A2.A.55b

Sketch right triangle *LMD*, $m\angle D = 90$, $m\angle L = 45$. Write your favorite number as the length of one of the sides. Using this information, find the lengths of the other two sides. Write the lengths as *exact* lengths. Do not use decimal approximations. Express the value of each of the following as the ratio of the sides of the triangle:

$\sin L$

$\cos L$

$\tan L$

$\sec L$

$\csc L$

$\cot L$

Reduce all of the fractions to lowest terms and compare your answers to another student's answers. What pattern do you see? What conclusions can you reach?

Fill in another number as the length of one of the sides. Compare your answers again. Based on what you have found, determine the exact values of each of the following:

$\sin 45^\circ$

$\cos 45^\circ$

$\tan 45^\circ$

$\sec 45^\circ$

$\csc 45^\circ$

$\cot 45^\circ$

A2.A.56 Know the exact and approximate values of the sine, cosine, and tangent of 0º, 30º, 45º, 60º, 90º, 180º, and 270º angles

A2.A.56a

Fill in the blanks in the following chart, giving *exact* numerical values.

θ (in radians)	0	$\frac{\pi}{6}$			$\frac{\pi}{2}$	π	
θ (in degrees)			45°	60°			270°
$\sin\theta$							
$\cos\theta$							
$\tan\theta$							

A2.A.56b
Fill in the blanks in the following chart.

Trigonometric Function	Exact Value	Approximate Value
$\sin \pi$		
$\tan 45^\circ$		
$\cos 270^\circ$		
$\sin \frac{\pi}{3}$		
$\cos$ ___	$\frac{\sqrt{3}}{2}$	
$\tan$ ___	$\sqrt{3}$	
$\cos$ ___	$\frac{\sqrt{2}}{2}$	

Under what circumstances would you prefer to find an approximation for each of these values, rather than giving an exact answer?

A2.A.57 Sketch and use the reference angle for angles in standard position

A2.A.57a
For each of the following angles, sketch the angle in standard position, then sketch its reference angle. Label the reference angle as α , and determine its measure.

150°

220°

$\frac{2\pi}{3}$

$\frac{5\pi}{4}$

A2.A.57b
For each of the following, use a reference angle to find the exact function value.
Include a sketch to show how you determined your answer.

$\sin 135^\circ$

$\tan 240^\circ$

$\cos \frac{7\pi}{6}$

$\sin \frac{7\pi}{4}$

A2.A.58 Know and apply the co-function and reciprocal relationships between trigonometric ratios

A2.A.58a
If $\cos\theta = \frac{4}{5}$, find $\sec\theta$.

A2.A.58b
If $\sin(x+20) = \cos(2x+10)$, find x.

A2.A.58c
What is the range of the function $f(x) = \sin x$? Based on your answer, what is the range of the function $f(x) = \csc x$? Explain your answer.

A2.A.59 Use the reciprocal and co-function relationships to find the value of the secant, cosecant, and cotangent of 0º, 30º, 45º, 60º, 90º, 180º, and 270º angles

A2.A.59a Fill in the blanks in the following chart, giving *exact* numerical values.

θ (in radians)			$\frac{\pi}{4}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$		
θ (in degrees)	0°	30°				180°	270°
$\sec\theta$							
$\csc\theta$							
$\cot\theta$							

A2.A.59b
Rewrite each of the following functions in terms of sine, cosine or tangent, and find the exact value:

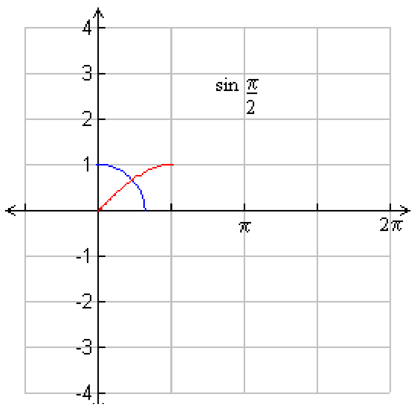
$$\sec\frac{\pi}{4}$$

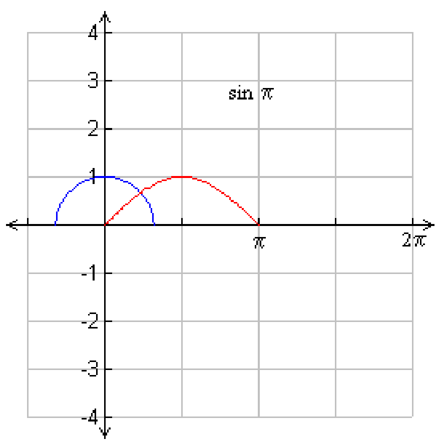
$$\csc 180^\circ$$

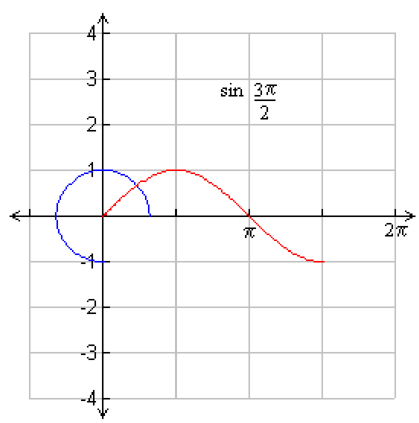
$$\cot\frac{\pi}{6}$$

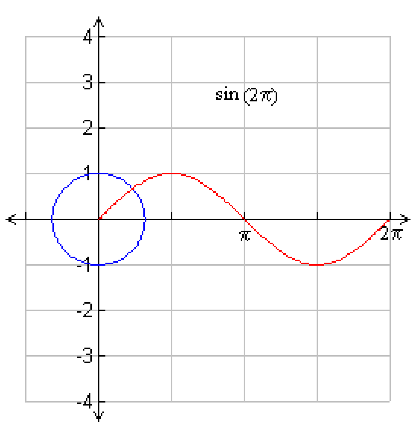
A2.A.60 Sketch the unit circle and represent angles in standard position

A2.A.60a
Each of the following graphs contains a graph of a portion of the unit circle and the corresponding portion of the curve $y = \sin(x)$. Utilize the two curves to determine each of the indicated function values.



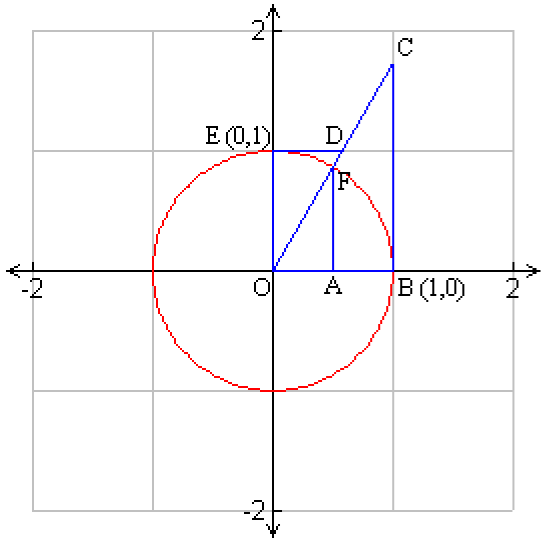






A2.A.60b

In the accompanying diagram, unit circle *O* has radii $\overline{OB}$, $\overline{OE}$, and $\overline{OF}$, $\overline{CB}$ is tangent to circle *O* at *B*, and $\overline{ED}$ is tangent to circle *O* at *E*. Points *O*, *F*, *D*, and *C* are collinear, and $\overline{FA} \perp \overline{OB}$.



If $m\angle COB = \theta$, name the line segment whose measure is each of the following:

$\sin\theta$

$\cos\theta$

$\tan\theta$

$\sec\theta$

$\csc\theta$

$\cot\theta$

A2.A.61 Determine the length of an arc of a circle, given its radius and the measure of its central angle

A2.A.61a

Find in centimeters the length of the arc intercepted by a central angle of 4 radians in a circle with a radius of 3.5 centimeters.

A2.A.61b

A sector has radius of 12 cm and angle 65°. To the *nearest tenth* of a centimeter, find its arc length.

A2.A.61c

A wheel has a radius of 3 feet. As the wheel turns, a rope connected to a five kilogram weight winds onto the wheel, causing the weight to move. If the wheel turns 135°, to the nearest foot, how far does the weight move?

A2.A.62 **Find the value of trigonometric functions, if given a point on the terminal side of angle θ**

A2.A.62a
Find the exact value of each of the following for angle θ in standard position if the point (4, -1) lies on its terminal side:

$\sin\theta$

$\cos\theta$

$\tan\theta$

$\cot\theta$

$\sec\vartheta$

$\csc\theta$

A2.A.62b
Find the exact value of $\sec\theta$ for angle θ in standard position if the point (-5, -4) lies on its terminal side.

A2.A.62c
The line $y = 3x$ creates an acute angle θ when it crosses the *x*-axis. Find the exact value of $\sec\theta$.

A2.A.63 **Restrict the domain of the sine, cosine, and tangent functions to ensure the existence of an inverse function**

A2.A.63a
In groups, sketch the graph of $y = \sin x$ from -4π to 4π.

Does the graph pass the vertical line test?
Now reflect $y = \sin x$ over the line $y = x$.
Is this a function? Why?
Find a section of the sine graph with a range of $-1 \le y \le 1$ and is a one-to-one function.
Reflect only the section with the restricted domain over the line $y = x$.
Does the new image pass the vertical line test?
State the domain and range of the image.

A2.A.64 **Use inverse functions to find the measure of an angle, given its sine, cosine, or tangent**

A2.A.64a
If the $\cot\theta = \frac{-8}{15}$ and $\cos\theta > 0$, find the exact value of $\sin\theta$.

A2.A.64b
Assuming an angle lies in Quadrant I, evaluate $\sin\left(\tan^{-1}\frac{5}{12}\right)$ as a fraction in lowest terms.

A2.A.65 **Sketch the graph of the inverses of the sine, cosine, and tangent functions**

A2.A.65a
Sketch the graph of $y = \tan x$ over the interval $-2\pi \le x \le 2\pi$.
Reflect the graph over the line $y = x$.
How would you restrict the domain to make the image a function?

A2.A.66 Determine the trigonometric functions of any angle, using technology

A2.A.66a
Using your calculator, find the value of each of the following to three decimal places:

$\tan 44^\circ\ 44'$

$\sin(-250^\circ\ 17')$

$\sec 31^\circ\ 42'$

$\cos\frac{7\pi}{9}$

$\csc\frac{5\pi}{9}$

A2.A.67 Justify the Pythagorean identities

A2.A.67a
Starting with $\cos^2 x + \sin^2 x = 1,$ and using your knowledge of the quotient and reciprocal identities, derive an equivalent identity in terms of $\tan x$ and $\sec x$. Show all your work.

A2.A.67b
Let $P(x, y)$ be a point in quadrant one on the unit circle, $x^2 + y^2 = 1.$ Let point *O* be the origin (0,0). Draw the line segment $OP.$ Let θ be the angle formed by $\overline{OP}$ and the positive portion of the *x*-axis. Now draw the perpendicular from *P* to meet the *x*-axis at point *M*.

State the ratio of $\frac{MP}{OP}$ in terms of $\theta.$

State the ratio of $\frac{OM}{OP}$ in terms of θ.

State the coordinates of point *P* in terms of θ.

Substitute your coordinates into the unit circle equation to verify one of the Pythagorean identities.

Now choose *P* in a different quadrant and repeat the process. Does the identity continue to be true?

A2.A.68 Solve trigonometric equations for all values of the variable from 0º to 360º

A2.A.68a
Solve for θ, to the *nearest* tenth of a degree, in the interval $0^\circ \le \theta \le 360^\circ$:

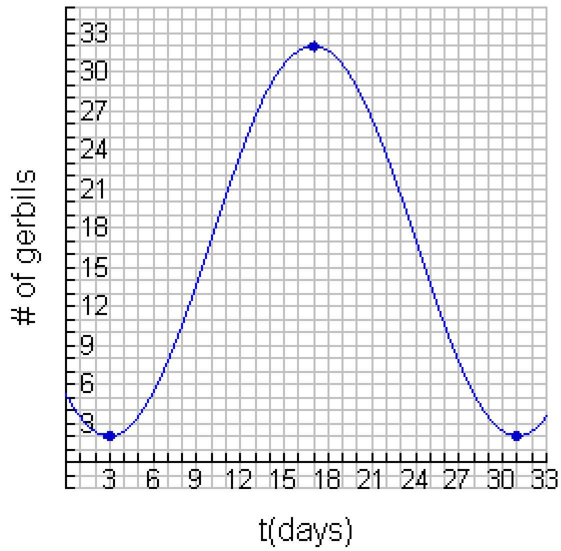
$$5\sin^2\theta + \sin\theta - 1 = 0$$

$$3\cos 2\theta = 2 - \sin\theta$$

$$2\cot\theta + \frac{4}{\cot\theta} = 7$$

A2.A.69 **Determine amplitude, period, frequency, and phase shift, given the graph or equation of a periodic function**

A2.A.69a
A pet store clerk noticed that the population in the gerbil habitat varied sinusoidally with respect to time, in days. He carefully collected data and graphed his resulting equation. From the graph, determine amplitude, period, frequency, and phase shift.



A2.A.69b
Given the following equations, determine the amplitude, period, frequency, and phase shift of each equation.

$$y = 2\sin\left(\frac{\pi}{3}(x-2)\right) - 4$$

$$y = -4 + 2\cos\left(\frac{\pi}{3}(x-3.5)\right)$$

Two students, Anthony and Chris, can be overheard discussing these equations. Anthony is certain that these equations are equivalent, while Chris is insisting they are different. Which student is correct? Explain your answer fully with graphs, tables, and a written paragraph supporting your position.

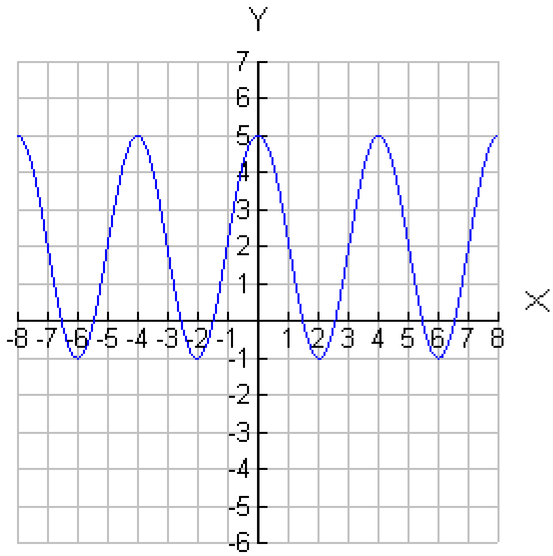
A2.A.70 **Sketch and recognize one cycle of a function of the form** $y = A\sin Bx$ **or** $y = A\cos Bx$

A2.A.70a
Sketch one complete cycle of each of the following.

$$y = 4\sin 3x$$

$$y = \frac{1}{2}\cos\left(\frac{\pi}{2}(x)\right)$$

A2.A.70b
Write both a sine and a cosine equation for the following graph.



A2. A.70c

The angle of inclination of the sun changes throughout the year. This changing angle affects the heating and cooling of buildings. The overhang of the roof of a house is designed to shade the windows for cooling in the summer and allow the sun's rays to enter the house for heating in the winter.

The sun's angle of inclination at noon in Albany, New York can be modeled by the formula:

$$\text{Angle of inclination (in degrees)} = -23.5\cos\left(\frac{360}{365}(x+10)\right)+47$$

where *x* is the number of days elapsed in the day of the year, with January first represented by $x=1$, January second represented by $x=2$, and so on.

Find the sun's angle of inclination at noon on Valentine's Day.

Sketch a graph illustrating the changes in the sun's angle of inclination throughout the year.

On what date of the year is the angle of inclination at noon the greatest in Albany, New York?

A2.A.71 **Sketch and recognize the graphs of the functions** $y=\sec(x)$, $y=\csc(x)$, $y=\tan(x)$, **and** $y=\cot(x)$

A2.A.71a
Sketch one cycle of each of the following equations. Carefully label each graph.

$y = \sec(x)$

$y = \csc(x)$

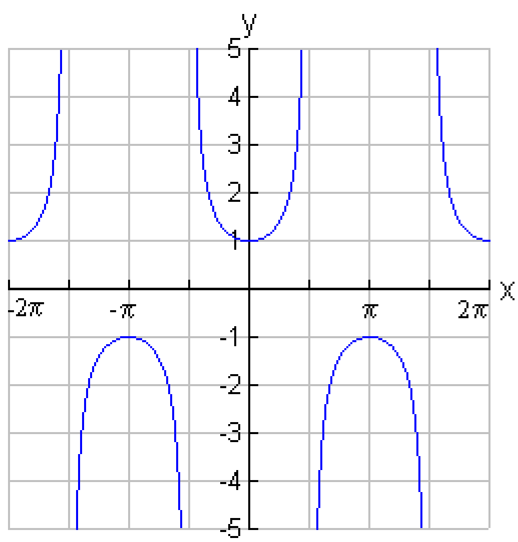
$y = \tan(x)$

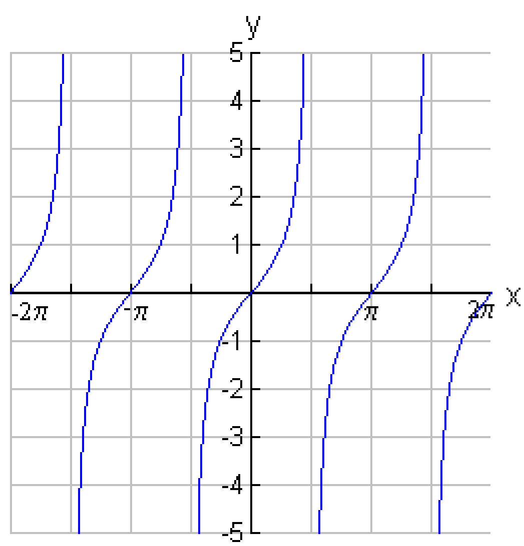
$y = \cot(x)$

Graph $y = \sec(x)$ and $y = \csc(x)$ at the same time on your calculator with a window of $0 \le x \le 2\pi,\ -4 \le y \le 4$. What conclusions can you make? Describe the similarities between the 2 functions.

Now, graph $y = \tan(x)$ and $y = \cot(x)$ at the same time on your calculator with a window of $0 \le x \le 2\pi,\ -4 \le y \le 4$. What conclusions can you make? Describe the similarities between the 2 functions.

A2.A.71b
Write an equation for each of the following graphs.

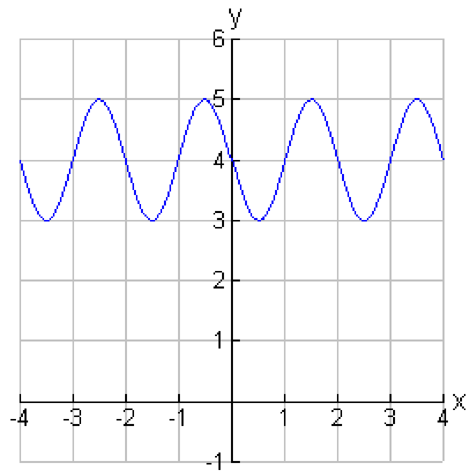


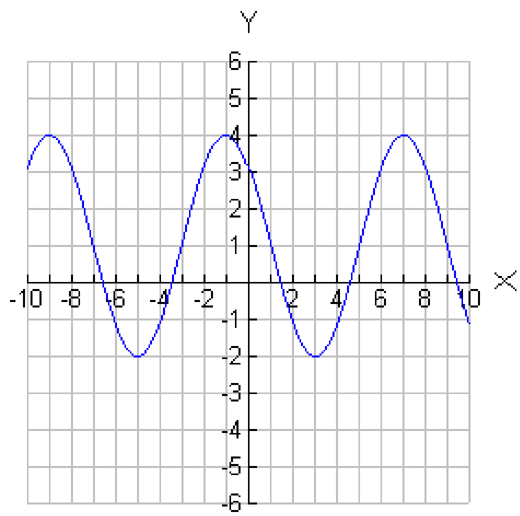


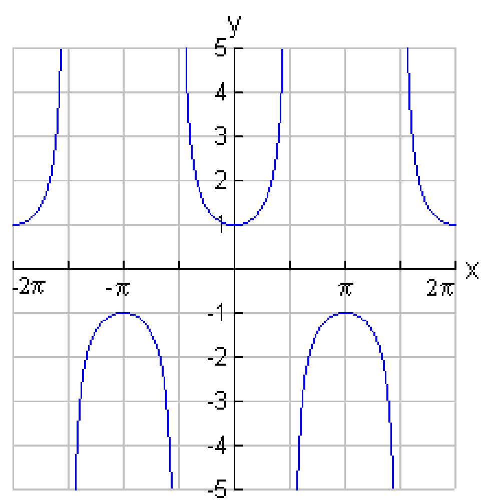
A2.A.72 Write the trigonometric function that is represented by a given periodic graph

A2.A.72a

Write a trigonometric function that matches each of the following graphs. Check your answers with a partner. If different equations have been obtained, confirm by graph or table, the accuracy of each equation.





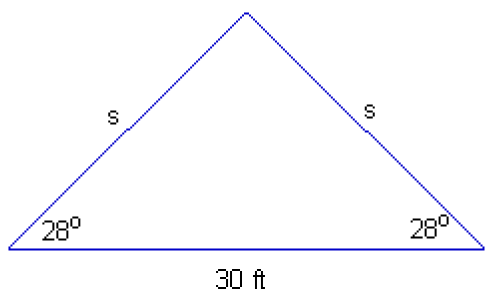


A2.A.72b
A team of biologists have discovered a new creature in the rain forest. They note the temperature of the animal appears to vary sinusoidally over time. A maximum temperature of 125° occurs 15 minutes after they start their examination. A minimum temperature of 99° occurs 28 minutes later. The team would like to find a way to predict the animal's temperature over time in minutes. Your task is to help them by creating a graph of one full period, an equation of temperature as a function over time in minutes, and a table of maximum, minimum, and average temperatures for the first 3 hours.

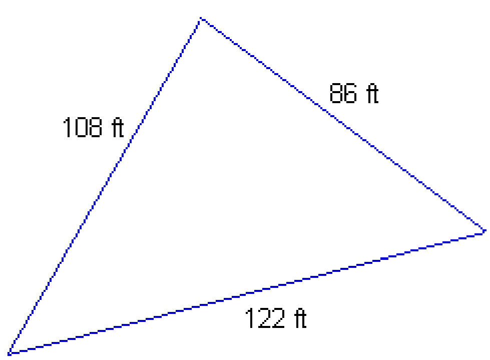
Discuss the advantages and disadvantages of each representation.

A2.A.73 Solve for an unknown side or angle, using the Law of Sines or the Law of Cosines

A2.A.73a
Chris is planning to build a new roof on his garage. He decides to pitch the sides of the roof at an angle of $28°$, and the width of the garage is 30 ft. Find the length of the sides of the roof, to the nearest tenth of a foot.



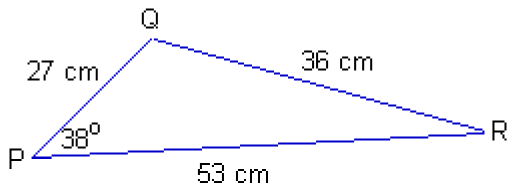
A2.A.73b
A surveyor measures the sides of a triangular plot of swamp land and labels the diagram. What is the largest angle in the triangle, to the nearest degree?



A2.A.73c

Two students in Ms. Baum's class worked on the problem below and got different solutions. Who got the correct answer? Explain your answer in detail.

Given triangle PQR, find the largest angle to the nearest degree.



Student #1

$$\frac{\sin(38°)}{36} = \frac{\sin(\angle R)}{27}$$
$$36\sin(\angle R) = 27\sin(38°)$$
$$m\angle R = \sin^{-1}\left(\frac{27\sin(38°)}{36}\right)$$
$$m\angle R = 27.5$$
$$m\angle Q = 180 - 38 - 27.5$$
$$m\angle Q = 115°$$

Student #2

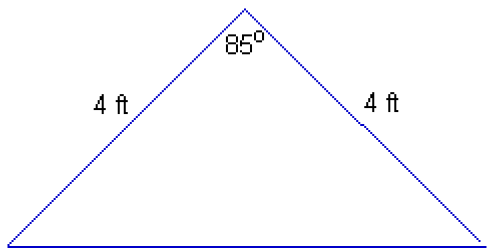
$$\frac{\sin(38°)}{36} = \frac{\sin(\angle Q)}{53}$$
$$36\sin(\angle Q) = 53\sin(38°)$$
$$m\angle Q = \sin^{-1}\left(\frac{53\sin(38°)}{36}\right)$$
$$m\angle Q = 65°$$

A2.A.73d

Write a word problem that illustrates an application of the Law of Sines, Law of Cosines, or a combination of both. Be creative, but be certain to use appropriate language and mathematical terminology when describing the problem situation. Also prepare a complete solution of the problem including a correctly labeled diagram and full mathematical explanation of how to solve the problem.

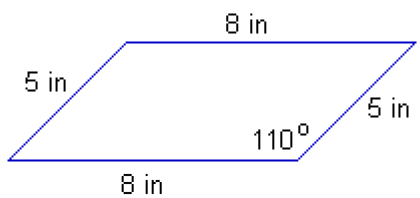
A2.A.74 **Determine the area of a triangle or a parallelogram, given the measure of two sides and the included angle**

A2.A.74a
Since Meisha's family has 3 people, her mother decided to make a triangular dinner table for them. She wanted to make sure that there was enough room for each of them to eat comfortably and wanted to determine the area of the table. Meisha's mother drew the design below to fit into their kitchen. Find the area of the table to the nearest square foot.



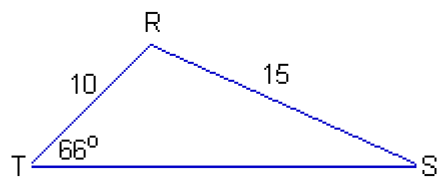
How long will the third side of the table be?

A2.A.74b
Samuel has decided to use stamped concrete for his backyard patio. He has chosen to make his design out of parallelograms. He has created a pattern and needs to find the area of his design. Using the following diagram, find the area to the nearest tenth of a square foot.



A2.A.75 Determine the solution(s) from the SSA situation (ambiguous case)

A2.A.75a
In the triangle below, find the missing angles and sides. Indicate the number of triangles which can be drawn with the given information. Round each measure to the nearest whole number.



A2.A.75b
Given triangle DEF, where $d = 24,\ e = 36,\ and\ \angle D = 25°$, find the missing angles and sides. Indicate the number of triangles which can be drawn with the given information. Round each measure to the nearest whole number.

A2.A.75c
Given triangle ABC, where $a = 17,\ b = 21,\ and\ \angle A = 64°$, find the missing angles and sides. Indicate the number of triangles which can be drawn with the given information. Round each measure to the nearest whole number.

A2.A.76 Apply the angle sum and difference formulas for trigonometric functions

A2.A.76a
If $\cos x = \frac{4}{5}$, and x is a positive acute angle, find $\sin\left(x + \frac{\pi}{2}\right)$.

A2.A.76b
If $\sin A = \frac{3}{5}$ and $\cos B = \frac{-12}{13}$ where $0 \le A \le \frac{\pi}{2}$ and $\pi \le B \le \frac{3\pi}{2}$ find:

$\cos(A + B)$

$\tan\ (A - B)$

A2.A.76c

Jenna's teacher has asked the class to find the exact value of sin(105 $^\circ$). Jenna's work is as follows.

$$\sin\left(105^\circ\right)=\sin\left(60^\circ+45^\circ\right)$$

$$\sin\left(60^\circ+45^\circ\right)=\sin\left(60^\circ\right)+\sin\left(45^\circ\right)$$

$$\sin\left(60^\circ\right)=\frac{\sqrt{3}}{2},\ \sin\left(45^\circ\right)=\frac{\sqrt{2}}{2}$$

$$\sin\left(105^\circ\right)=\frac{\sqrt{3}}{2}+\frac{\sqrt{2}}{2}\ =\ \frac{\sqrt{3}+\sqrt{2}}{2}$$

Jenna's teacher has marked this as incorrect. Using counterexamples or an indirect proof, demonstrate why Jenna's work is not correct.

A2.A.77 Apply the double-angle and half-angle formulas for trigonometric functions

A2.A.77a

If $\sin x=\frac{4}{5}$, what is $\cos 2x$?

A2,A.77b

Using the half-angle formula, find the exact value of $\cos 15^\circ$.

A2.A.77c

Given that $\tan x=\frac{a}{b}$, and $\pi\le x\le\frac{3\pi}{2}$ evaluate in terms of *a* and *b*:

$\sin 2x$

$\tan\ 2x$

Back to top

Measurement

Students will determine what can be measured and how, using appropriate methods and formulas.

Units of Measurement

A2.M.1 Define radian measure

A2.M1a

Anthony's teacher told the class that a unit circle has a circumference of 2π . This confuses him because he thought a circle has 360° . Write a detailed explanation that compares degrees to radians. Be as thorough as possible in order to help Anthony understand the connection, including diagrams and equations.

A2.M.2 **Convert between radian and degree measures**

A2.M2a
Convert each radian measure to degree measure.

$\frac{\pi}{3}$ radians, $-\frac{2\pi}{5}$ radians, 4 radians

A2.M2b
Convert each degree measure to radian measure.

$120°,\ -245°,\ 470°$

A2.M2c
In which quadrant, or on what axis, does the terminal side of each angle lie?

$\frac{4\pi}{3}$

$-\frac{5\pi}{4}$

$\frac{9\pi}{2}$

[Back to top]

Statistics and Probability

Students will collect, organize, display, and analyze data.

Collection of Data

A2.S.1 **Understand the differences among various kinds of studies (e.g., survey, observation, controlled experiment)**

A2.S.1a
Search in newspapers and magazines and then write a brief but thorough description of a study that could be characterized as:

a survey
an observational study
a designed experiment

A2.S.2 **Determine factors which may affect the outcome of a survey**

A2.S.2a
Randomization is an important characteristic of a well-designed survey. Name two sources of bias that can be eliminated by randomization.

A2.S.2b
It has been decided that 78 people need to be surveyed to decide the public's opinion on a school building project. A student suggests that they survey the first 78 people who enter the school.

Do you think that this proposed way of sampling is an unbiased way to perform the survey or can you describe a better way to achieve a fair and accurate response?

A2.S.3 Calculate measures of central tendency with group frequency distributions

A2.S.3a
The table displays the frequency of scores on a twenty point quiz. The mean of the quiz scores is 18.

Score	15	16	17	18	19	20
Frequency	2	4	7	13	k	5

Find the value of k in the table. Find the mode and the median of all the quiz scores .

A2.S.3b
The table displays the number of uncles of each student in a class of Algebra 2 .

uncles	0	1	2	3	4	5
students	2	5	4	6	10	8

Find the mean, median, and mode of the uncles per student for this data set.

A2.S. 3c
The average earning of 110 juniors for a week was \$35, while during the same week 90 seniors averaged \$ 50. What were the average earnings for that week for the combined group?

A2.S.4 Calculate measures of dispersion (range, quartiles, interquartile range, standard deviation, variance) for both samples and populations

A2.S.4a
For the data set: {5, 4, 2, 5, 9, 3, 4, 5, 3, 1, 6, 7, 5, 8, 3, 7} construct the five-number summary. Find the range and the interquartile range of the data set.

A2.S.4b
Find to the nearest tenth the standard deviation of the distribution:

score	1	2	3	4	5
frequency	14	15	14	17	10

A2.S.4c
If the five numbers { 3, 4, 7, x, y} have a mean of 5 and a standard deviation of $\sqrt{2}$. Find x and y given that $y > x$.

A2.S.5 Know and apply the characteristics of the normal distribution

A2.S.5a
Five hundred values are normally distributed with a mean of 125 and a standard deviation of 10.
Answer the following questions:
What percent of the values is in the interval 115-135?
What percent of the values is in the interval 100-150?
What interval about the mean includes 50% of the data?
What interval about the mean includes 95% of the data?

A2.S.5b
A machine is used to fill plastic soda bottles. The amount of soda dispensed into each bottle varies slightly. Suppose the amount of soda dispensed into the bottles is normally distributed. If at least 99% of the bottles must have between 585 and 595 milliliters of soda, find the greatest standard deviation, to the nearest hundredth, that can be allowed.

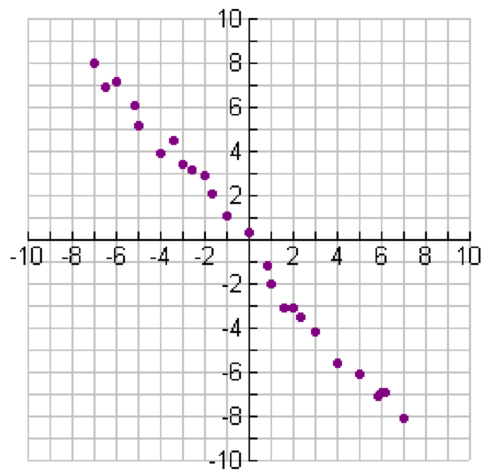
A2.S.5c
The lifetime of a battery is normally distributed with a mean life of 40 hours and a standard deviation of 1.2 hours. Find the probability that a randomly selected battery lasts longer than 42 hours.

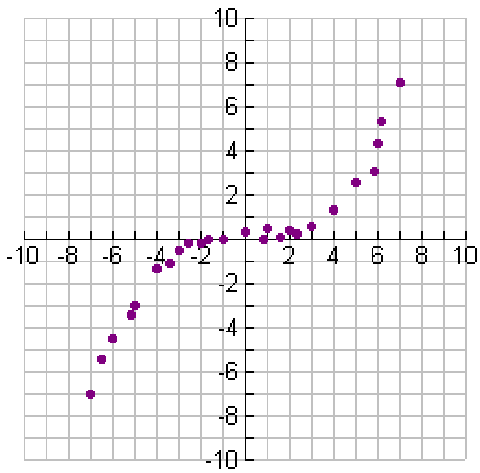
Students will make predictions that are based upon data analysis.

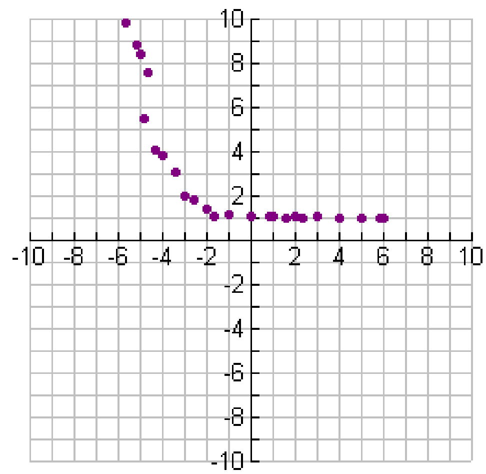
Predictions from Data

A2.S.6 Determine from a scatter plot whether a linear, logarithmic, exponential, or power regression model is most appropriate

A2.S.6a
Which type of function (linear, exponential, logarithmic, or power) would best model the data in each of the scatter plots shown below? Explain your reasoning.







A2.S.6b
Sketch a scatter plot whose regression model could be logarithmic and one whose regression model could be exponential. Use your knowledge of exponential and logarithmic functions to explain the differences and similarities in the two scatter plots.

A2.S.7 Determine the function for the regression model, using appropriate technology, and use the regression function to interpolate and extrapolate from the data

A2.S.7a
Take a cup of very hot water, and measure the temperature of the water. Place the cup in a safe place, and measure the temperature every five seconds for a total of one minute. Record this information in your notebook.

Enter the information in two lists on your calculator. In the first list, enter the time, in seconds, since the experiment began. In the second list, enter the temperature of the water for each indicated time.

Make a scatter plot, where the independent variable is time, in seconds, and the dependent variable is temperature. What type of regression function would best model this data? Why?

Write the regression equation with all constants rounded to the nearest thousandth.
Using this equation, determine the temperature of the water after 25 seconds has passed.
Determine the temperature of the water after two hours has passed. Do both of these answers seem logical to you? Why?

(N.B. If you have a temperature probe for your calculator or computer, it may be used to conduct this experiment.)

A2.S.7b
Stretch your arms out as wide as you can. Have someone measure the length, in inches, of your arm span, from the tip of your fingers on one hand, across your back, to the tip of your fingers on your other hand. Have the person also measure your height, in inches. Share this information with your classmates, and aggregate all of the data.

Enter the information in two lists on your calculator. In the first list, enter the heights, in inches, of all of the participants. In the second list, enter the arm span, in inches, of all of the participants.

Make a scatter plot, where the independent variable is height, in inches, and the dependent variable is arm span, in inches. What type of regression function would best model this data? Why?

Write the regression equation with all constants rounded to the nearest tenth.
Using this equation, determine the arm span of a person who is 5 feet tall.
Research to see what other measurements would form similar relationships.

A2.S.7c
The table shows the amount of a medicine for treating a disease in the bloodstream over the 9 hours following a dose of 10 mg. It seems that the rate of decrease of the drug is approximately proportional to the amount remaining.

Time(hours)	0	1	2	3	4	5	6	7	8
Drug Amount (mg)	10	8.3	7.2	6.0	5.0	4.4	3.7	2.8	2.5

Use this information to find a suitable function to model this data.

Using your model, when will there be less than 1 mg. of the medicine in the patient's blood stream?

If the initial dose was 15 mg, when would the amount of the medicine in the bloodstream fall below 5 mg?

A2.S.7d
According to the National Weather Service, the 2005 average monthly temperature in degrees Fahrenheit at Central Park in New York City is given below:

Month	Jan.	Feb.	March	April	May	June	July	Aug.	Sept.	Oct.	Nov.	Dec.
Average Temperature (°F)	31.3	36.5	39.4	55.1	58.9	74.0	77.5	79.7	73.3	57.9	49.6	35.3

Write a sinusoidal function that models the average monthly temperature, using $t = 1$ to represent January. According to your model, what is the average temperature in December?

Explain the discrepancy from your model to the average monthly temperature in December.

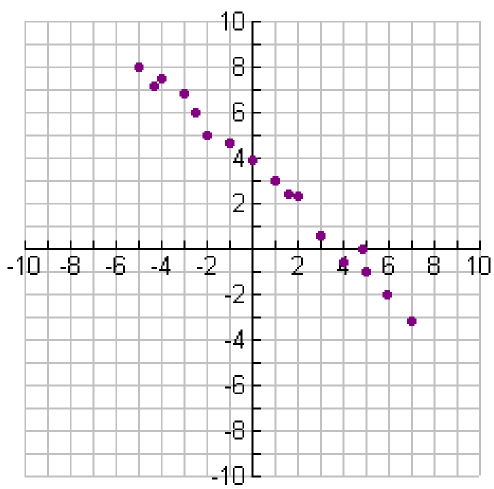
How could the model be improved?

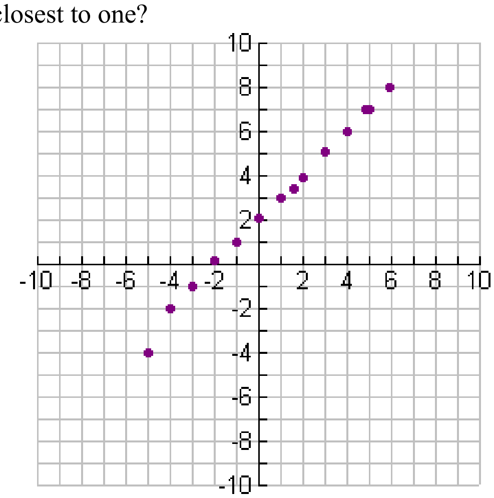
A2.S.8 Interpret within the linear regression model the value of the correlation coefficient as a measure of the strength of the relationship

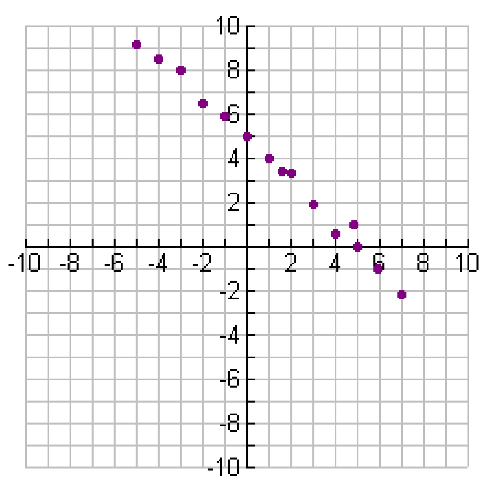
A2.S.8a
Which of the following scatter plots would have a positive linear correlation coefficient?

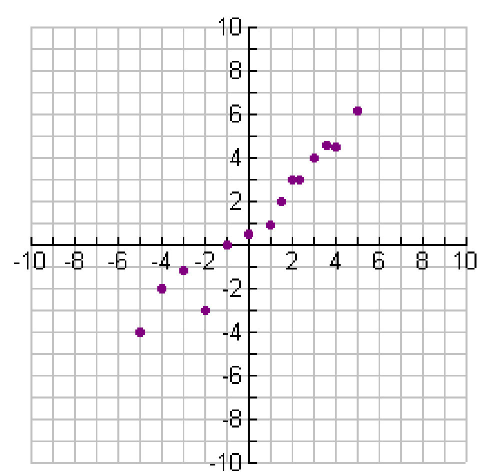
Which would have a negative linear correlation coefficient?

Which would have a linear correlation coefficient closest to one?









A2.S.8b
Use two thermometers, one Celsius and the other Fahrenheit, to measure the temperature of your hand. Place both thermometers in the palm of your hand and gently squeeze your hand shut. (If you have a temperature probe for your calculator or computer, it may be used instead.)

Gather this information with your classmates, and aggregate all of the data.

Enter the information in two lists on your calculator. In the first list, enter the Celsius temperatures. In the second list, enter the Fahrenheit temperatures.

Make a scatter plot, where the independent variable is degrees Celsius, and the dependent variable is degrees Fahrenheit.

Find the linear regression equation that best fits the data.

What is the correlation coefficient? Why would you expect its value to be close to one?

Students will understand and apply concepts of probability.

Probability

A2.S.9 Differentiate between situations requiring permutations and those requiring combinations

A2.S.9a
Determine whether each of the following situations would require calculating a permutation or a combination. Explain your answer.

Selecting a lead and an understudy for a school play
Assigning students to their seats on the first day of school
Selecting three students to represent the school at an Honor Society conference in Washington, D.C.

A2.S.9b
Describe a situation that would require calculating a permutation, and another situation that would require calculating a combination. Share your thoughts with a classmate and discuss the strategies used to determine when to calculate a permutation and when to calculate a combination. Write a brief summary of your conclusions and be prepared to discuss your ideas with the class.

A2.S.10 Calculate the number of possible permutations $\left({}_nP_r\right)$ of *n* items taken *r* at a time

A2.S.10a
Evaluate each of the following

$${}_6P_3$$

$${}_5P_5$$

$${}_8P_1$$

$${}_4P_2 \cdot {}_5P_3$$

$$\frac{{}_7P_4}{3!}$$

A2.S.10b
A teacher is making a multiple-choice quiz. She wants to give each student the same questions, but have each student's questions in a different order. If there are twenty-seven students in the class, what is the least number of questions the quiz must contain?

A2.S.11 Calculate the number of possible combinations ($_nC_r$) of *n* items taken *r* at a time

A2.S.11a
Calculate each of the following:

$_7C_2$

$_{10}C_{10}$

$_6C_5 \cdot {}_6C_3$

$_8C_1$

$$\frac{8!}{3!(8-3)!}$$

A2.S.11b
A coach must choose five starters from her team of twelve players. How many different ways can she choose the starters?

A2.S.12 Use permutations, combinations, and the Fundamental Principle of Counting to determine the number of elements in a sample space and a specific subset (event)

A2.S.12a
The local Family Restaurant has a daily breakfast special in which the customer may choose one item from each of the following groups:

Breakfast Sandwiches	**Accompaniments**	**Juice**
egg and ham egg and bacon egg and cheese	breakfast potatoes apple slices fresh fruit cup pastry	orange cranberry tomato apple grape

How many different breakfast specials are possible? How many different breakfast specials without meat are possible?

A2.S.12b
How many three-digit numbers can be formed from the digits 0,1,2, …, 9? How many of these numbers would be multiples of five?

A2.S.12c
There are fourteen juniors and twenty-three seniors in the Service Club. The club is to send four representatives to the State Conference. How many different ways are there to select a group of four students to attend the conference? If the members of the club decide to send two juniors and two seniors, how many different groupings are possible?

A2.S.12d Find the number of arrangements that can be made from the letters of each of the following words:

THINK
MATHEMATICS

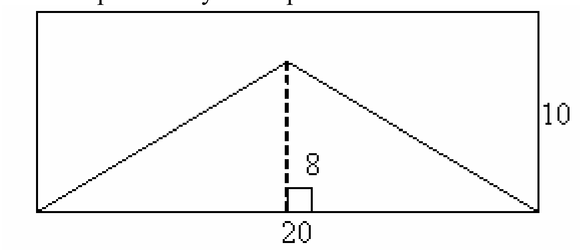
How many arrangements would there be if only consonants are used?

A2.S.13 Calculate theoretical probabilities, including geometric applications

A2.S.13a
Each of the five members of the Music Committee for the spring dance is allowed to select one song to be played by the DJ at the dance from a list of forty approved songs. What is the probability that at least two committee members select the same song?

A2.S.13b
A bag contains three chocolate, four sugar, and five lemon cookies. Greg takes two cookies from the bag for a snack. Find the probability that Greg did not take two chocolate cookies from the bag. Explain why using the complement of the event of not choosing two chocolate cookies might be an easier approach to solving this problem.

A2.S.13c
Find the probability that a point selected at random will lie inside the triangle below:



A2.S.14 Calculate empirical probabilities

A2.S.14a
When a paper cup is tossed onto a table, it can land one of three ways: open end up, bottom up, or on its side. Give each student in class a paper cup. Have each student toss their cup onto a table thirty times, and record the result of each of their tosses. Then tally the results of the entire class's tosses.

Using the class's results of the experiment, compute the following empirical probabilities: P(open end up), P(bottom up), P(landed on side).

Compute the following theoretical probabilities: P(open end up), P(bottom up), P(landed on side). How do the empirical probabilities found compare to the theoretical probabilities? Tape a penny to the bottom of the cup. Repeat the experiment. What effect do you think this will have on the outcomes of the tosses? Compute the same empirical and theoretical probabilities as you did the first time. How do these result compare to your prediction?

A2.S.14b
Use a random number generator to generate five hundred random numbers which will simulate tossing a fair coin thirty times. Let an even number represent the result of the coin coming up heads, and an odd number represent the result of tails.

Generate a second list of five hundred random numbers to represent the second toss of a fair coin. Use the two lists of random numbers to compute the following empirical probabilities: P(head, tail), P(both the same), P(both different).

Compute the following theoretical probabilities for the same experiment: P(head, tail), P(both the same), P(both different). How do the empirical and theoretical probabilities compare?

A2.S.15 Know and apply the binomial probability formula to events involving the terms *exactly*, *at least*, and *at most*

A2.S.15a
If a binomial experiment has seven trials in which the probability of success is p and the probability of failure is q, write an expression that could be used to compute each of the following probabilities: P(exactly five successes), P(at least five successes), P(at most five successes).

A2.S.15b
Experience has shown that $\frac{1}{200}$ of all CDs produced by a certain machine are defective. If a quality control technician randomly tests twenty CDs, compute each of the following probabilities: P(exactly one is defective), P(half are defective), P(no more than two are defective).

AS.S.15c
If a fair coin is tossed ten times, what is the probability that it lands heads up: exactly six times, at most three times, at least seven times?

A2.S.16 Use the normal distribution as an approximation for binomial probabilities

A2.S.16a
Give an example of an experiment where it is appropriate to use a normal distribution as an approximation for a binomial probability. Explain why in this example an approximation of the probability is a better approach than finding the exact probability.

A2.S.16b
Estimate the probability of getting at least fifty-four heads in one-hundred tosses of a fair coin.

A2.S.16c
A fifty-question multiple-choice test consists of questions which each have four possible choices for the answer to each question. Estimate the probability of answering at least 70% of the questions correctly if all of the answers are random guesses.

Back to top